Practice Questions to Review for the MA 15300 Final

Bring the following items to the final:
- ✓ **your graphing calculator**
- ✓ **Number 2 pencils**
- ✓ **ID Number** (You can get this after logging into **http://go.pfw.edu**. It is on the Home page.) You can also get it from your instructor.)

The final exam will evaluate how well you meet the course goals of MA 15300:
- Highlight the link of mathematics to the real world.
- Develop a wide base of mathematical knowledge, including
  - basic skills and concepts,
  - a functional view of mathematics, including graphical, analytical, numerical, and contextual viewpoints (Note: using these four representations is the *Rule of Four*),
  - properties and applications of some of the basic families of functions,
  - geometric visualization,
  - problem solving, predicting, critical thinking, and generalizing.
- Incorporate the use of general academic skills such as
  - communicating mathematics concepts,
  - understanding and using technology.

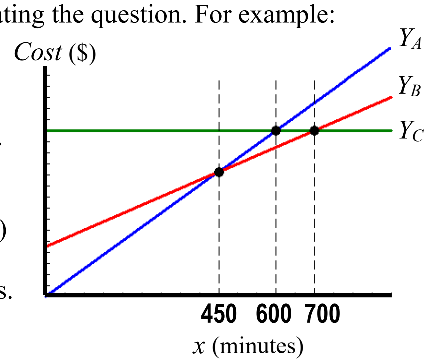
Just like the chapter exams throughout the semester, this exam tests your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.

Format of the exam: The actual final exam will consist of both multiple-choice questions and open ended (constructed response) questions. Include units in your answers whenever appropriate. You may certainly use your calculator (but not its manual). In fact, some questions will *require* a graphing calculator.

NO formula sheets, notes, books, or other external sources may be used.

For the open ended questions, you will need to show all of your work. If you are basing your reasoning on a graph, then sketch a labeled graph, with numerical values on the axes. If you base your reasoning from a table, you must include the table, which consists of at least five sets of entries. If using an equation, write out the steps you used to solve the equation. Remember you the solution to the problem is not simply the *end result*, but showing the *process* that you used to justify your claim.

Be careful when explaining your reasoning. This is more than simply restating the question. For example: Three long distance phone plans are represented by the graphs shown, where x gives the number of minutes used per month and Y_A, Y_B, and Y_C are the monthly charges, in dollars, using Company A, B, and C, respectively. Which plan is best? Explain your reasoning using the graphs.



Incorrect Answer: "Plan A is cheapest because it gives the lowest price."
(This essentially restates the question! Plus, it's wrong.)

Best Answer: "Plan A is cheapest if you use less than 450 minutes.
Plan B is cheapest if you use between 450 and 700 minutes.
Plan C is cheapest if you use more than 700 minutes."

The graph of Y_A is below the graphs of Y_B and Y_C for $x < 450$.
The graph of Y_B is below the graphs of Y_A and Y_C for $450 < x < 700$.
The graph of Y_C is below the graphs of Y_A and Y_B for $x > 700$.

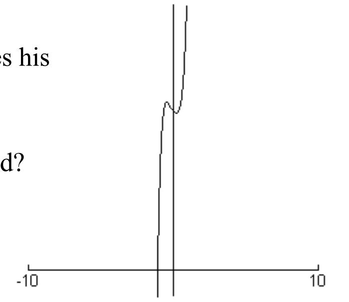
How to prepare for the exam: Some of the practice questions that follow are from previous final exams over this material. Note that the exam will NOT look exactly like these questions, so you should also review previous homework assignments, *eHW*, quizzes, and tests, as well as material worked on during class meetings. **Topics from the last chapter on *Polynomial and Rational Functions* will receive more of an emphasis than earlier chapters.** Keep the *Rule of Four* in mind when solving problems, just as you have done throughout the semester.

Worked out solutions to all of these problems are posted on your instructor's Blackboard Site or at www.pfw.edu/departments/coas/depts/math/courses/math-153.html (The MA 15300 Course Website).

Sample Questions for the Final Exam

1. Suppose you and Charlie are working together in a group to determine the long run behavior of $f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$. Charlie uses his graphing calculator in the window $-10 \le x \le 10$ and $-10 \le y \le 100$ and sees the graph shown. Charlie concludes that the long run behavior is as follows:

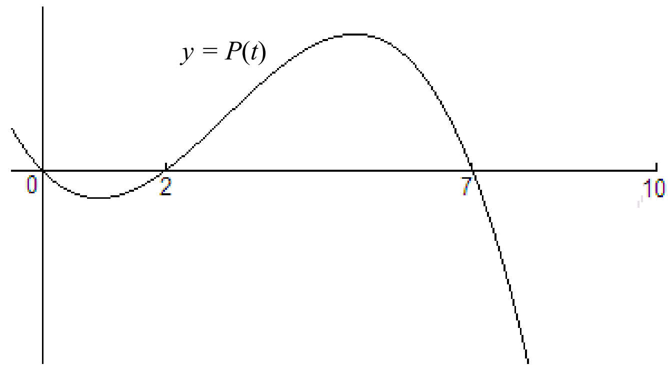
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to \infty$. How should you respond?



$-10 \le x \le 10 \quad -10 \le y \le 100$

A. "Good job, Charlie!"
B. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$."
C. "Sorry, Charlie!
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to -\infty$."
D. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to -\infty$."
E. "Sorry, Charlie!
As $x \to 0^+$, then $y \to -\infty$; as $x \to 0^+$, then $y \to \infty$."

For **Questions 2-3**, $P(t)$ is a polynomial of degree 3 whose graph is shown.



2. For $0 \le t \le 10$, $P(t)$ describes the temperature of a certain chemical reaction in degrees Celsius, t seconds after the reaction began. Suppose the temperature reached –1 degree Celsius exactly 1 second after the reaction began.

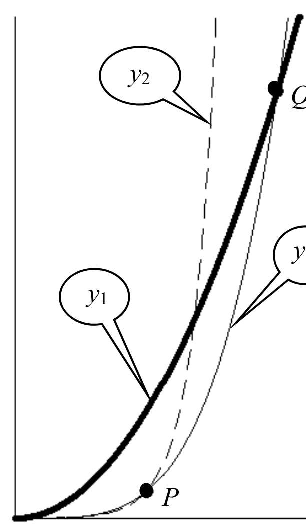
Determine the formula for $P(t)$.
Then find the minimum temperature that the chemical reaction reaches in the first 10 seconds after it began.

A. $-1^\circ C$ B. $-10^\circ C$ C. $-20^\circ C$ D. $-40^\circ C$ E. None of these

3. A certain power function $Q(t)$ has the same long run behavior as $P(t)$, so much that $Q(t)$ and $P(t)$ look nearly indistinguishable if you graph both of these functions with technology and zoom out for very large values of t. This tells us that it would not be sensible to use $P(t)$ to model the temperature of the reaction for *all* $t \ge 0$. What is the formula for $Q(t)$?

A. $Q(t) = -t^3$ B. $Q(t) = t^3$ C. $Q(t) = -\frac{1}{6}t^3$ D. $Q(t) = \frac{1}{6}t^3$ E. None of these.

4. Graphs of $y = 64x^2$, $y = x^4$, and $y = 4^x$ are shown. Assume the graph shows their entire **long run behavior** for $x \ge 0$.



(a) Which function corresponds to which?

A. $y_1 = 64x^2$	$y_2 = x^4$	$y_3 = 4^x$
B. $y_1 = x^4$	$y_2 = 64x^2$	$y_3 = 4^x$
C. $y_1 = 4^x$	$y_2 = 64x^2$	$y_3 = x^4$
D. $y_1 = 4^x$	$y_2 = x^4$	$y_3 = 64x^2$
E. None of these		

(b) Use the table feature of a graphing calculator to report the integer coordinates of the intersection points P and Q.
(c) Report the **nonnegative** values of x which solve $y_1 \ge y_3$.
(d) Report all values of x which solve $y_1 \ge y_3$.

Questions 5-7

The EDI pharmaceutical company has recently acquired the abandoned but historic Rotting Hill building and has decided to move its employees into this renovated building one month at a time. The table gives the number, $E(t)$, of EDI employees who have moved to the Rotting Hill building t months after the building was acquired.

t	$E(t)$
0	0
1	30
2.02	48
2.83	60

5. The data for $E(t)$ is modeled by a power function.
Find the formula for $E(t)$. Which of the following would be closest to the value of $E(7)$?
A. 100 B. 110 C. 120 D. 130 E. 210

6. Unfortunately, many of the employees of EDI who have their offices located in the Rotting Hill building have contracted a mysterious disease which incapacitates them for weeks at a time. The table gives the number, $S(t)$, of EDI employees who have their offices located in the Rotting Hill building *and* are sick t months after the initial acquisition of the building.

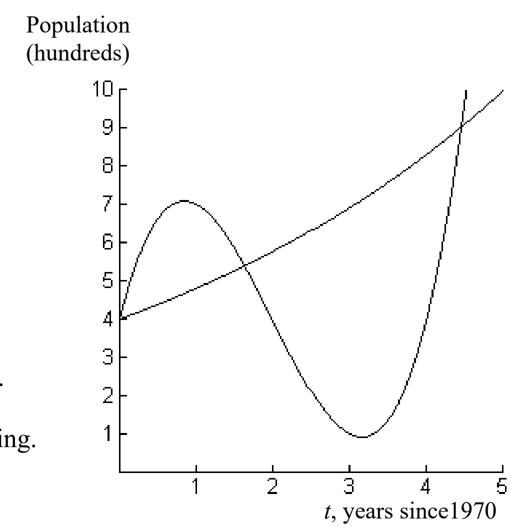
t	$S(t)$
0	0
1.05	6
2.05	15
2.98	25

The data for $S(t)$ is modeled by a power function. Find the formula for $S(t)$.
Which of the following would be closest to the value of $S(11)$?
A. 30 B. 70 C. 110 D. 120 E. 150

7. When the ratio of number of sick employees in a building to total number of employees in a building is greater than 0.75 the building is declared to have sick building syndrome and is closed down for health inspection. How many months after the Rotting Hill building is first acquired by EDI will it be closed for health inspection? Select the one closest to your answer.
A. 10 days B. 2 months C. 5 months D. 7 months E. 16 months

8. The population (in hundreds) of the town *Polynomia* grows according to $P(t) = t^3 - 6t^2 + 8t + 4$, where $t = 0$ corresponds to January 1, 1970.

The population (in hundreds) of the town *Exponentia* is given by $E(t)$, where again $t = 0$ corresponds to January 1, 1970. The town initially has 4 hundred people when $t = 0$ and it increases by 20% each year.



The graph shows the populations over the first five years.
If the population follows these mathematical models, which of the following must be true?
Select the best response.

A. The population of *Polynomia* is always more than 90 people.

B. After 1974 the populations of both towns are always increasing.

C. The population graphs will intersect a total of four times.
The population of *Exponentia* will overtake and exceed the population of *Polynomia* sometime after the year 2028.

D. Both A and B are true.

E. All of the above are true.

Questions 9-10

The volume of pollutants (in millions of cubic feet) in Smirch Reservoir is given by

$$P(t) = 360 + 9t$$

where t is in years. The total volume of Smirch Reservoir (which includes both pollutants and water and also in millions of cubic feet) is gradually increasing and is given by

$$R(t) = 12{,}000 + 12t$$

Let $C(t)$ be the fraction of the reservoir's total volume that consists of pollutants.

Write an expression for $C(t) = \dfrac{P(t)}{R(t)}$ in terms of t and use your expression to answer the questions below.

9. In year $t = 0$, what percent of the reservoir's total volume consists of pollutants?

A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$ % E. None of these

10. According to the mathematical model, if these trends were to continue for many, many years, about what percentage of Smirch Reservoir's total volume would eventually consist of pollutants?

A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$ % E. None of these

11. Find any zeros. Report any noninteger zeros exactly.

a) $f(x) = 400x(6x^2 - 42)$

b) $f(x) = -\frac{1}{100}(x-2)(x^3 + 3x^2)(x+1)^2$

c) $f(x) = 10x^3 - 4x^4 - 4x^2$

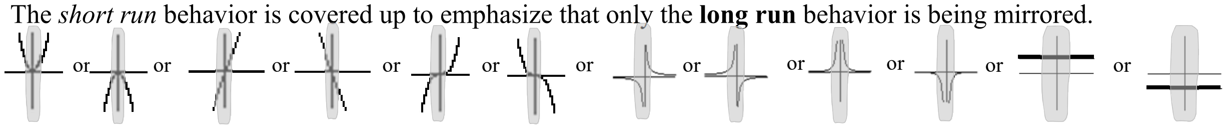
12. Report the leading term, the leading coefficient, and the degree for each polynomial.

a) $f(x) = 400x(6x^2 - 42)$

b) $f(x) = -\frac{1}{100}(x-2)(x^3 + 3x^2)(x+1)^2$

c) $f(x) = 10x^3 - 4x^4 - 4x^2$

13. For each of the rational functions $f(x)$ below, **i.)** Report the formula of the power function $g(x)$ which has the same **long run** behavior as $f(x)$. **ii)** sketch the power function $g(x)$. Pick from one of the choices below. The *short run* behavior is covered up to emphasize that only the **long run** behavior is being mirrored.



Note: $g(x)$ and $f(x)$ will look nearly indistinguishable if you graph both of these functions with technology and zoom out for very large values of x. **iii)** Finally, report the equation of the horizontal asymptote of $f(x)$, if there is one. If none, state so.

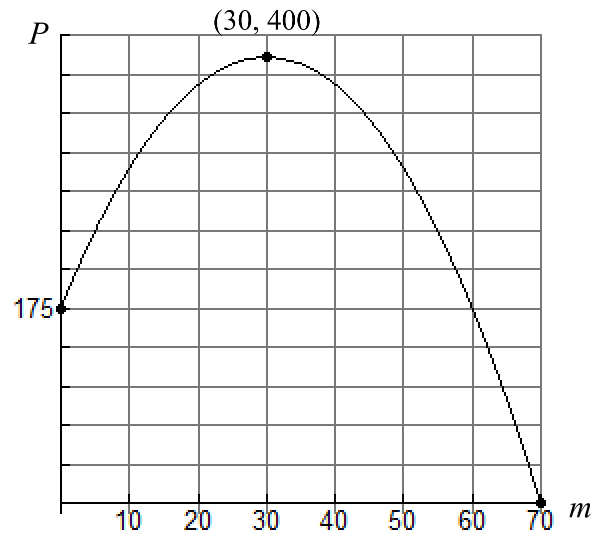
a) $f(x) = \dfrac{400x(6x^2 - 42)}{10x^3 - 4x^4 - 4x^2}$ **b)** $f(x) = \dfrac{400x(6x^2 - 42)}{10x - 4x^3 - 4x^2}$ **c)** $f(x) = \dfrac{400x^2(6x^2 - 42)}{10x - 4x^3 - 4x^2}$

Questions 14-21

Peter grows peppers. The yield, P, of peppers (in pecks) that Peter picks is a function of the amount, m, of fertilizer (in pounds) used, which is given by $P = f(m)$. See the graph below.

14. The statement $f(30) = 400$ means
A. The yield ranges from 30 to 400 pecks of peppers.
B. When 30 lb of fertilizer is applied, the yield is a maximum of 400 pecks of peppers.
C. For every 30 lb of fertilizer added to the orchard, you increase the yield by 400 pecks.
D. When 400 lb of fertilizer is applied, the yield is 30 pecks of peppers.
E. You apply 30 to 400 pounds of fertilizer to the orchard.

15. The vertical intercept for the graph represents:
A. The maximum yield of the orchard.
B. The amount of fertilizer that must be applied to produce a maximum yield.
C. The yield without applying any fertilizer at all.
D. The initial amount of fertilizer applied to the orchard.
E. The amount of fertilizer that will kill all the trees and produce no yield at all.



16. Estimate the range.
A. $0 \le f(m) \le 70$
B. $175 \le f(m) \le 400$
C. $70 \le f(m) \le 175$
D. $70 \le f(m) \le 400$
E. $0 \le f(m) \le 400$

17. For what values of m is the function increasing?
A. $175 < m < 400$
B. $0 < m < 400$
C. $0 < m < 30$
D. $30 < m < 70$
E. None of these

18. For what values of m is the function concave up?
A. $175 < m < 400$
B. $0 < m < 400$
C. $0 < m < 30$
D. $0 < m < 70$
E. None of these

19. For what values of m is $P > 175$?
A. $175 < m < 400$
B. $60 < m < 400$
C. $60 < m < 70$
D. $0 < m < 60$
E. None of these

20. The function $f(m)$ is quadratic. Report the following:
a) the ***equation*** of the parabola's axis of symmetry
b) the formula for $f(m)$ in **vertex form**.

21. Write the formula for $f(m)$ in **factored form**.

22. In the year 1900 the population P of a town was 1160 people but it grew by 10 people every year.
In the year 1900 the population Q of a town was 1000. The town grew by 1.13% every year.
Find when the population of Q overtakes the population of P.
Select the response which is closest to the answer.
A. 1.3 years B. 39 years C. 70.9 years D. 116 years E. Q will never overtake P.

Questions 23-24

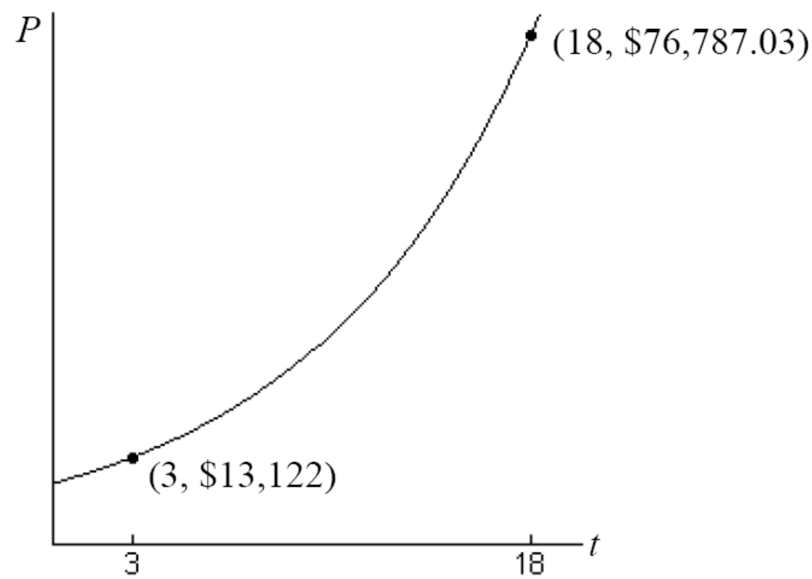
The amount Q of drug present in a person's body is $Q = 20(0.4)^t$,
where Q is in milligrams at time t, and t is in hours.

23. What percent of the drug is lost per hour?
A. 4% B 20% C. 40% D. 6% E. 60% F. 80%

24. What is the growth factor?
A. 0.4 B. 4.0 C. 6.0 D. 20 E. 60 F. 80

Questions 25-26

The graph gives the balance, P, of an investment in year t. Find a possible formula for $P = f(t)$ assuming the balance grows exponentially.



25. Which amount is closest to the initial balance?
A. $5,424
B. $5,454
C. $9,216
D. $10,122
E. $11,664
F. $41,812

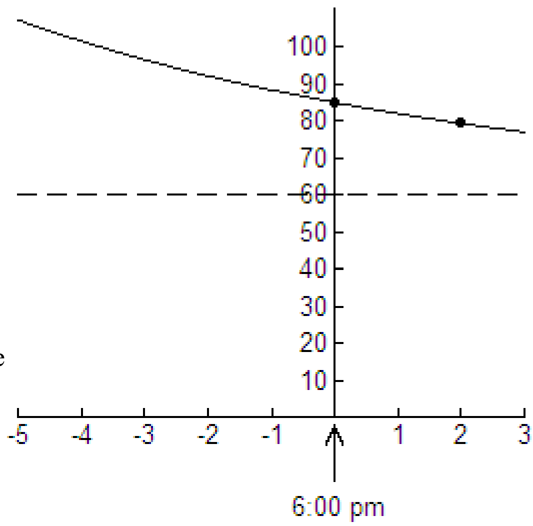
26. What annual interest rate does the account pay?
A. 1.125%
B. 11.25%
C. 12.5%
D. 34%
E. 112.5%
F. 125%

27. Good news! ☺: You win a contest to have dinner at the home of NCIS* director Mark D. Clookie!
Bad news! ☹: When given a tour of the premises, his butler is discovered in the wine cellar, sprawled dead on the floor.

Special Agent Clookie has with him a temperature probe, which he uses to find the body temperature of the butler. At 6:00 pm, he determines that the body temperature is 85ºF.
Two hours later, after dinner, he takes another temperature reading to find the body has cooled to 79.36ºF.

(a) Clookie shares that he keeps his wine cellar at a constant temperature of 60ºF.
Because of this, butler's body temperature will decay exponentially**.
He sketches the graph shown on a paper napkin.
The graph has the model $y = ab^t + 60$.
Give the constants a and b.
Which is true? Select one:

A. $a = 25$ G. A and D M. C and D
B. $a = 60$ H. A and E N. C and E
C. $a = 85$ I. A and F O. C and F
D. $b \approx -2.82$ J. B and D P. None of these
E. $b \approx 0.478$ K. B and E
F. $b \approx 0.88$ L B and F



(b) Another possibility is to use the equation $y = He^{kt} + 60$.
Give the constants H and k.
Which is true? Select one:

A. $H = 25$ G. A and D M. C and D
B. $H = 60$ H. A and E N. C and E
C. $H = 85$ I. A and F O. C and F
D. $k \approx -1.04$ J. B and D P. None of these
E. $k \approx -0.74$ K. B and E
F. $k \approx -0.128$ L B and F

(c) Clookie recalls that his niece restocked his wine cellar earlier that day. House records show that she arrived at 2:45 p.m. It is also known that his butler had a reputation of good health - his body temperature was 98.6ºF. Was the butler already dead when his niece delivered the wine? When was the time of death? Explain your reasoning.

* NCIS is the Naval Criminal Investigative Service
**The effect to which Clookie refers is known as *Newton's Law of Cooling*.

28. The monthly charge for a waste collection service is \$32 for 100 kilograms of waste and is \$48 for 180 kilograms of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the number of kilograms of waste, w.
(b) What is the slope of the line found in part **(a)**?
Give units and interpret your answer in terms of the cost of waste collection.
(c) What is the vertical intercept of the line found in part **(a)**?
Give units and interpret your answer in terms of the cost of waste collection.

29. A research facility on an island off of Costa Rica has 900 gallons of fresh water for a two-month period.

(a) There are 4 members of the research team and each is allotted 3 gallons of water per day for cooking and drinking. Find a formula for $f(t)$, the amount of fresh water left on the island after t days has elapsed, assuming that each member of the team uses their total allotment of water each day.

(b) Evaluate and interpret the following expressions

(i) $f(0)$

(ii) $f^{-1}(0)$

U.S. dollars	Shillings
$1.00	2200
$2.50	5500
$3.00	6600

30. In a second hand clothing store in Kampala, Uganda, the table* shown is used to exchange U.S. dollars for shillings. The function is linear.

(a) What is the y-intercept of the graph of this function?

(b) A pair of trousers cost 4000 shillings. How much is this in U.S. dollars?

(c) If you have $4.00 U.S, can you afford a coat which sells for 8500 shillings?

*Based on a December 2005 Associated Press report.

31. A summer amusement park, charges $10.00 for admission. An average of 10,000 people visit the park each day it is open. Consultants predict that for each $1.00 increase in the entrance price, the park would lose an average of 500 daily customers.

(a) Construct a table of values which shows the entrance price, p, and number of tickets sold, N. Your table should have at least five entries.

(b) Let $N = f(p)$. Find a formula for this function.

(c) Add a third column to your table in part **(a)** which gives the daily revenue, R, for each entrance price p. (The *revenue* is the total amount received by the park before any costs are deducted.)

(d) Let $R = g(p)$. Find a formula for this function.

(e) Find the axis intercepts of $N= f(p)$. Interpret what each means to the staff at the amusement park.

(f) Find and interpret the axis intercepts of the revenue function $R = g(p)$.

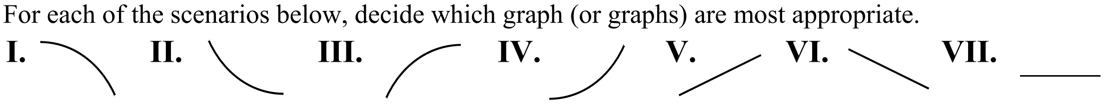
(g) What ticket price maximizes the revenue?

(h) Sketch graphs of $N = f(p)$ and $R = g(p)$ on the same set of axes. Be sure your sketch is properly labeled with the values found in parts **(e)**, **(f)**, and **(g)**.

32. Use inverse properties and properties of logs to simplify the expression $e^{x\ln a}$. (Circle one.)

A. x^a B. e^{ax} C. $\frac{a}{x}$ D. ax E. a^x F. None of these

33. For each of the scenarios below, decide which graph (or graphs) are most appropriate.



(a) "Even though the child's temperature is still rising, the penicillin seems to be taking effect."

(b) "Your distance from the Atlantic Ocean, in kilometers per minute, increases at a constant rate."

(c) "The interest on your savings plan is compounded annually.

(d) "At first your balance grows slowly, but its rate of growth continues to increase."

(e) "The annual profit is decreasing at a higher rate each year."

(f) The function has a positive rate of change and the rate of change is decreasing.

(g) "The price of memory chips isn't decreasing as quickly it used to be."

(h) The function is concave down.

(i) The function is decreasing.

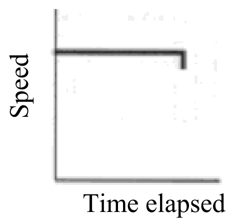
(j) The function is constant.

(k) The average rate of change of the function is constant.

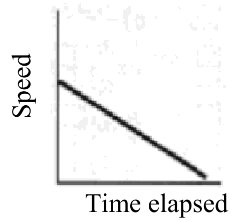
34. Indicate which graph matches the statements. Note the choice of axes.

a. A train pulls into a station and lets off passengers.

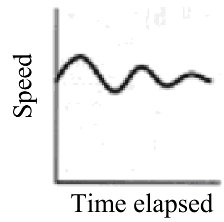
I.



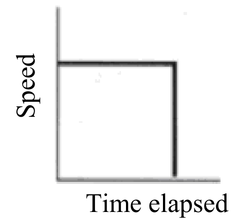
II.



III.

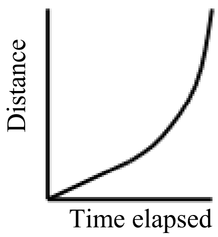


IV.

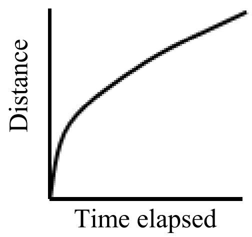


b. I start to walk to class at a slow steady rate. I hear the clock chimes and walk faster and faster.

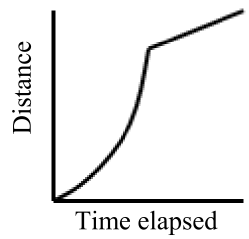
I.



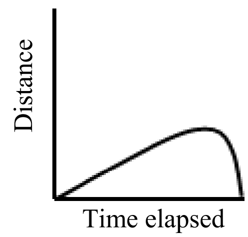
II.



III.

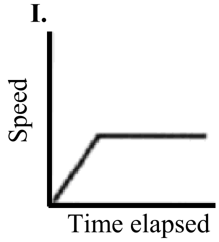


IV.

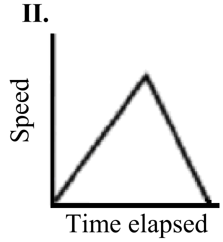


c. I climb a hill at a steady pace and then start to run down one side.

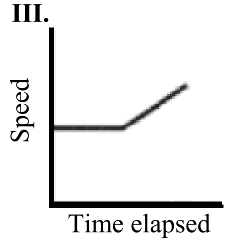
I.



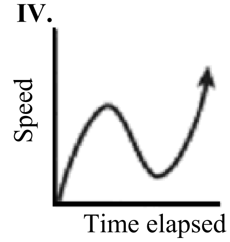
II.



III.

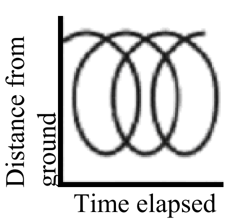


IV.

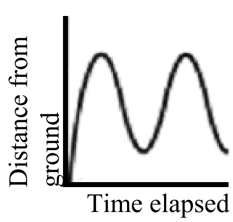


d. I ride on a Ferris Wheel.

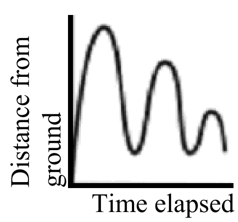
I.



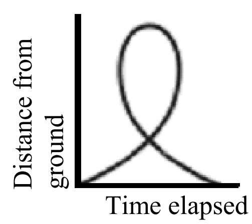
II.



III.

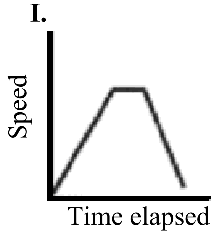


IV.

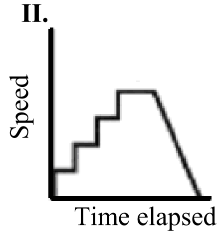


e. A child climbs up a slide and then slides down.

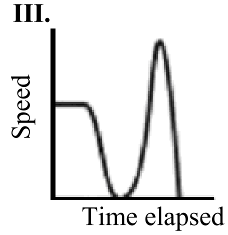
I.



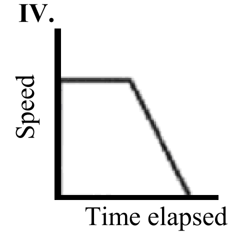
II.



III.



IV.



35. One description best fits each function. Decide which one, and write its letter in the corresponding blank.

i. _____ $P(t) = 300 - 2t$ ii. _____ $Q(t) = 300e^{0.02t}$

iii. _____ $R(t) = 300(0.98)^t$ iv._____ $S(t) = -\frac{4}{3}t^2 + 300$

A. The population, which began at 300, declines at a constant rate, becoming extinct in 15 years.
B. The population increases exponentially at first and then levels off.
C. The population, which began at 300, is growing at the continuous rate of 2 percent each year.
D. In one year, 98 percent of the population is lost.
E. The population, which was originally at 300, has been increasing at a rate of 98 percent.
F. The population starts at 300 and has dropped to 250 after 25 years.
G. The population, which began at 300, decreases faster and faster.
H. The population, originally at 300, has been decreasing at the annual rate of 2 percent.
I. The population, which was originally at 300, undergoes explosive logarithmic growth, increasing at the annual rate of 2 percent.

36. The path of an artillery shell, in feet, fired from a military base is given by $h(x) = 0.96x - 0.004x^2$.

a) What is the **exact** maximum height, in feet, of the artillery shell?
b) How many feet from the military base is the shell when it hits the ground?
c) Report the vertex.
d) Report the ***equation*** of the parabola's axis of symmetry.
e) Write the function in **vertex form**.
f) Write the function in **factored form**.

37. Short Questions:

(a) Give the equations of all vertical and horizontal asymptotes of $y = \dfrac{8x^2 - 8}{2x^2 - 4x}$

(b) Give the domain of $f(x) = \sqrt{x - 100}$.

(c) A substance decays according to the formula $P(t) = 200(0.5)^{t/17}$, where t is in minutes. What is the half-life? What percent of the substance decays each minute? (Report integer answers for each.)

(d) The doubling time of a population is 12 years. What is its *tripling* time? (Report to the nearest year.)

(e) Show that $x = 2$ is a solution to the equation $4x + 8 = 4^x$. Then find all values of x which solve $4x + 8 > 4^x$. (Report your answer accurate to 3 decimal places.)

(f) True or False: $\log(A + B) = \log A + \log B$.

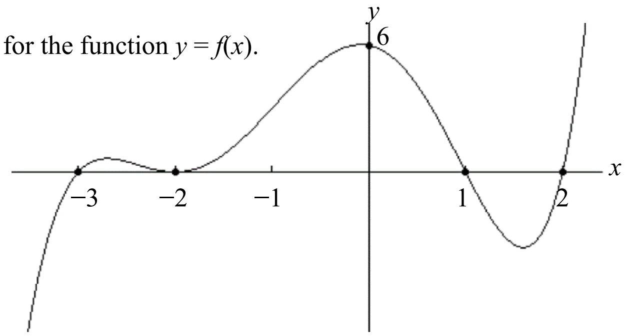
38. Which of the following functions has its domain identical with its range?

A. $f(x) = x^2$ B. $g(x) = \log x$ C. $h(x) = x^3$ D. $i(x) = |x|$ E. None of these.

39. For the graphs in this problem assume all global (or long run) behavior is shown.

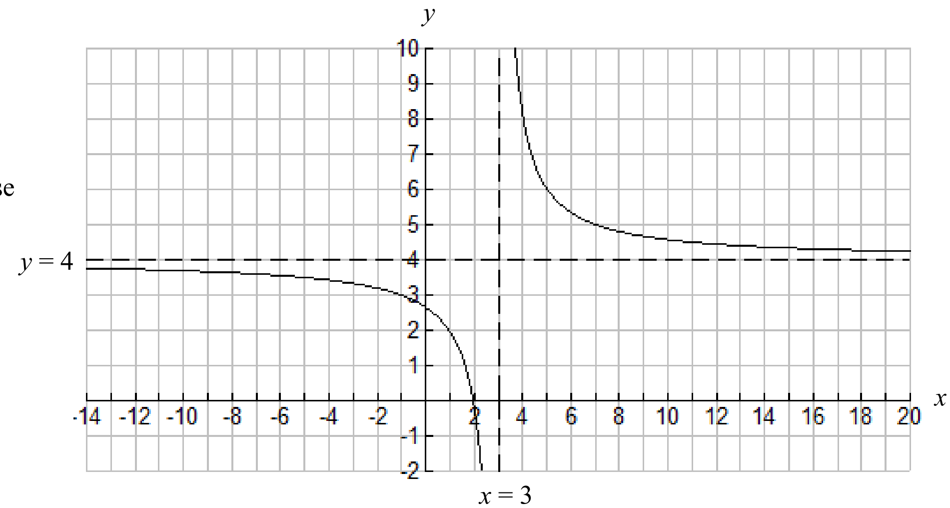
a. Find a possible formula of least possible degree for the function $y = f(x)$. Then use your formula to find $f(3)$.

A. 30
B. 75
C. 306
D. 501
E. None of these



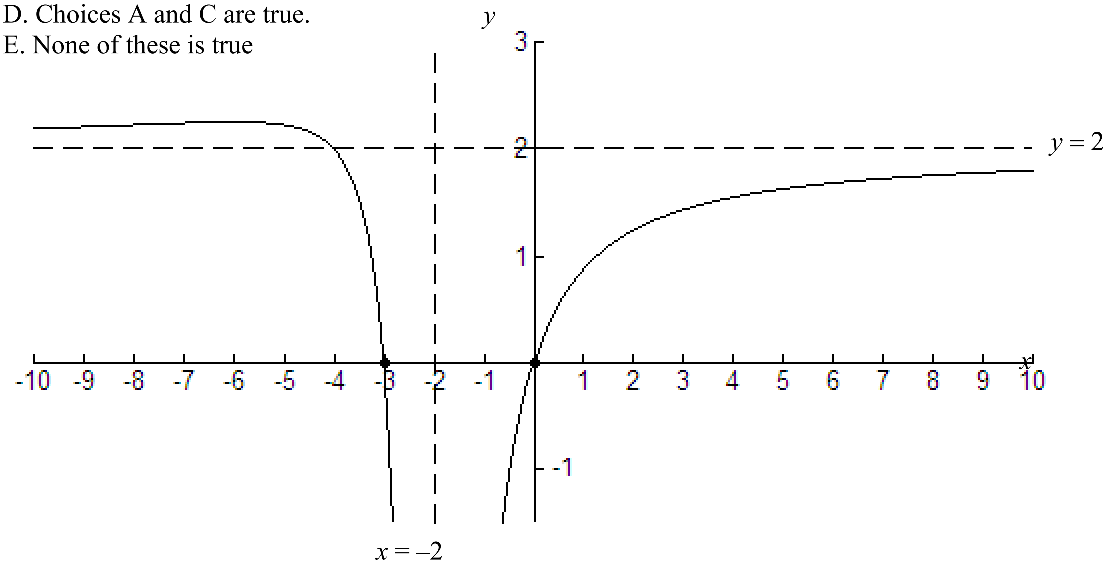
b. Find the formula for $f(x)$. It has a single zero at 2 and asymptotes shown. Use your formula to find $f(403)$. Tip: Use a table.

A. 4.1
B. 4.3
C. 4.01
D. 4.03
E. None of these

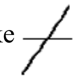
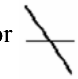
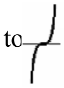
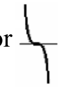
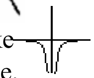
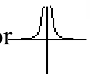


c. Find the formula for $f(x)$. It has zeros and asymptotes shown.
Using your formula, determine which of the following must be true? Tip. Use a table.

A. $f(-1) = -4$
B. $f(1) = 1$
C. $f(-6) = 2.25$
D. Choices A and C are true.
E. None of these is true



40. A rational function $y = f(x)$ has the following properties:

- there is only one zero at 4,
- the short run behavior near that zero looks like  or  (as opposed to  or )
- there is one vertical asymptote at $x = 2$,
- the short run behavior near the vertical asymptote looks like  or 
- the degree of the denominator is the lowest degree possible,
- there is a horizontal asymptote of $y = 0$, and
- $f(0) = -8$

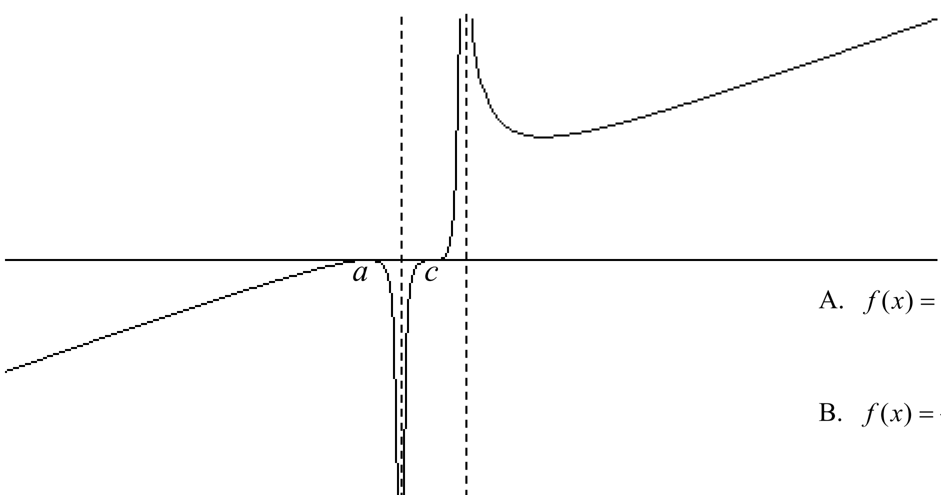
Find the formula for $f(x)$. Then use your formula to find $f(3)$.

A. -16
B. -8
C. -4
D. 8
E. 16

41. Assume a, b, c, and d are positive real numbers.
The rational function $f(x)$ graphed below has the following properties:

- <u>short run behavior</u>:
  - zeros are at a, c
  - vertical asymptotes are at $x = b$ and $x = d$
- <u>long run behavior</u>:
  as $x \to -\infty, y \to -\infty$
  as $x \to \infty, y \to \infty$
  Consequently, there is no horizontal asymptote.

Assume k is some positive real number. Which could be its equation?



A. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^4}$

B. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^2}$

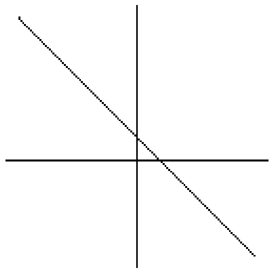
C. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)}$

D. $f(x) = \dfrac{k(x-a)^4(x-c)^3}{(x-b)^3(x-d)^4}$

E. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)(x-d)^3}$

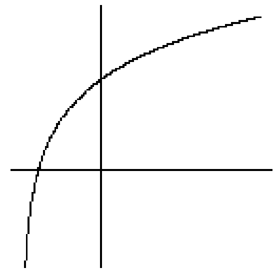
42. For each of the graphs below, select the formula beneath the graph which best fits the behavior of the graph. In each case, assume that A, B, and C are positive real numbers. (Circle your choice.)
Scales are not shown on the axes, so the graphs may not have a true geometric perspective.

(I)



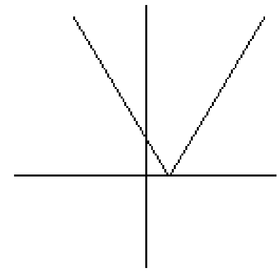
(a) $y = Ax + B$
(b) $y = -Ax - B$
(c) $y = B - Ax$
(d) $y = \dfrac{x+A}{x+A}$

(II)



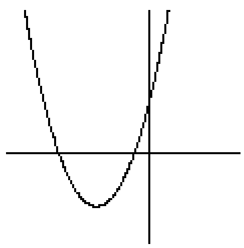
(a) $y = e^{-Ax}$
(b) $y = \log(x - A)$
(c) $y = \log(x + A)$
(d) $y = A^{(x+B)}$

(III)



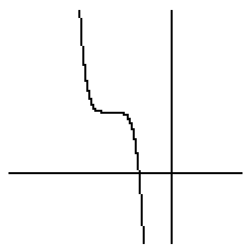
(a) $y = |x - A|$
(b) $y = |x + A|$
(c) $y = |x| - A$
(d) $y = |x| + A$

(IV)



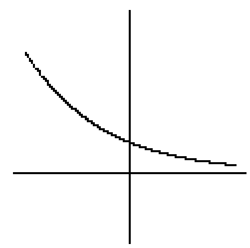
(a) $y = Ax^2 - B$
(b) $y = C - A(x+B)^2$
(c) $y = A(x+B)^2 - C$
(d) $y = A(x-B)^2 - C$

(V)



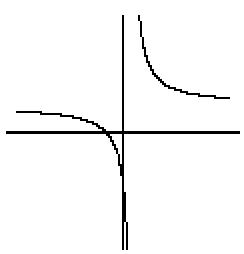
(a) $y = -Ax^5 + B$
(b) $y = Ax^3 + B$
(c) $y = -A(x+B)^5 + C$
(d) $y = A(x+B)^5 + C$

(VI)



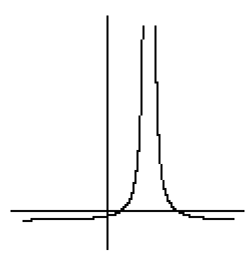
(a) $y = -\ln(x + A)$
(b) $y = -(1/A)^x$
(c) $y = -A^x$
(d) $y = (1/A)^x$

(VII)



(a) $y = \dfrac{A(x-B)}{x+C}$
(b) $y = -\dfrac{A(x-B)}{x+C}$
(c) $y = \dfrac{A(x+B)}{x-C}$
(d) $y = \dfrac{-A(x+B)}{x-C}$

(VIII)



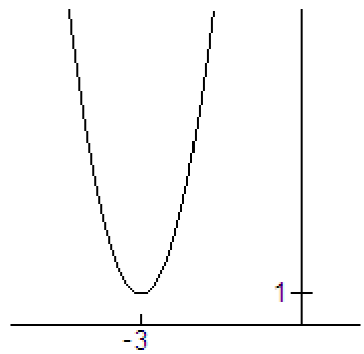
(a) $y = \dfrac{A}{(x-B)^2} - C$
(b) $y = \dfrac{A}{(x+B^2)} - C$
(c) $y = \dfrac{A}{(x-B)} - C$
(d) $y = \dfrac{-A}{(x-B)} - C$

43. Water is pumped into a swimming pool at a constant rate. The table shows the volume of water every 30 minutes after the pumping began. What is the average rate of change?
A. 75 gallons B. 0.4 minutes per gallon
C. 0.4 gallons per minute D. 2.5 minutes per gallon
E. 2.5 gallons per minute

Time, t (min)	Volume, V (gal)
30	1075
60	1150
90	1225
120	1300

44. The graph of the function is a translation of $y = 5x^2$, shifted left 3 and up 1. What is the **range** of the graph?

A. all real numbers
B. $y \geq 1$
C. $y \geq -3$
D. $y \geq -1$
E. $y \leq 1$



45. Assuming x, y, and w are positive real numbers, which of the following is $\log \frac{x^3 y^2}{\sqrt{w}}$?

A. $x^3 + y^2 - \sqrt{w}$ B. $\frac{1}{3}\log x + \frac{1}{2}\log y - 2\log w$ C. $3\log x + 2\log y - \frac{1}{2}\log w$

D. $\dfrac{3\log x + 2\log y}{\frac{1}{2}\log w}$ E. None of these

46. Solve for x to the nearest hundredth: $25^x = 3^{600}$
(Most calculators are unable to solve this numerically or graphically due to overflow problems.)
A. 409.56 B. 530.44 C. 204.78 D. No solution E. None of these

47. Find the vertex of the parabola: $y = 4x^2 + 8x + 100$.
A. $(-2, 100)$ B. $(1, 130)$ C. $(0, 100)$ D. $(-1, 96)$ E. None of these

48. Find all the zeros of the polynomial function: $f(x) = 7(x^3 - 3x^2 - 4x)$.
A. $-1, 4$ B. $-4, 1$ C. $-1, 0, 4$ D. $-1, 0, 4, 7$ E. None of these

49. Find all possible values of x for which $9x^2(x+6)(x-6)^2 \geq 0$.
Support your reason graphically.
A. $-6 \leq x \leq 6$ B. $-6 \leq x \leq 0$ or $x \geq 6$ C. $x \geq -6$ D. $x \leq 6$ E. None of these

50. An initial deposit of \$4000 is made in a savings account for which the interest is compounded continuously. If the interest rate is 7.3%, how long will it take, to the nearest 0.01 year, for the investment to triple? Use $A = Pe^{rt}$.
A. 0.15 years B. 2.79 years C. 6.54 years D. 15.05 years E. None of these

51. Simplify $\ln\left(\frac{1}{\sqrt{e^x}}\right)$

A. $\frac{1}{x}$ B. $-x$ C. $-\frac{x}{2}$ D. $\frac{1}{\sqrt{x}}$ E. None of these

52. Which of the following is true about the graph of $y = f(x) = b^x$? List **all** correct answers.

I. It increases if $b > 1$
II. It decreases if $b < 0$
III. It has y-intercept (0, 1) if $b > 0$.

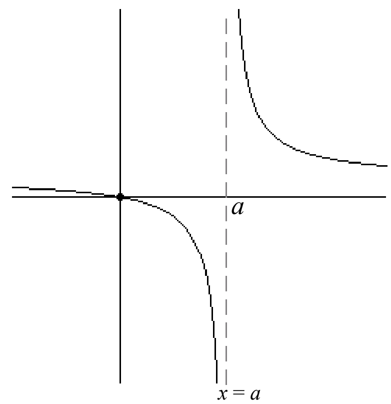
A. I, II and III B. I and II C. II and III D. I and III E. III only.

53. Use what you know about transformations and the graph of $y = \log x$ to answer the following about the graph of $f(x) = 2 + \log(x - 1)$. Which are true? List **all** correct answers.
The graph of $f(x) = 2 + \log(x - 1)$

I. increases for all values of x in its domain.
II. crosses the x-axis at 1
III. never touches the y-axis
IV. passes through the point (2, 2).

Note: Don't be misled by technology when answering this question.
A. I, II and III B. I and II C. II and IV D. I and IV E. I, III and IV only.

54. A function passes through the origin and has a vertical asymptote at $x = a$, where $a > 0$. It has the graph shown. Which could be its equation?



A. $f(x) = \dfrac{1}{x-a}$ B. $f(x) = \dfrac{1}{x+a}$ C. $f(x) = \dfrac{x}{x-a}$

D. $f(x) = \dfrac{x}{x+a}$ E. $f(x) = \dfrac{x}{(x-a)^2}$

55. Let a be some constant.

Which is true about $f(x) = \dfrac{2ax}{(x-a)^2}$?

A. Its horizontal asymptote is $y = 2a$.
B. Its horizontal asymptote is $y = 2$.
C. Its horizontal asymptote is $y = \dfrac{2a}{x}$.
D. Its horizontal asymptote is $y = 0$.
E. It has no horizontal asymptote.

56. The relationship of pH to the hydrogen ion concentration, C, is pH = $-\log C$.
If the pH is 2.1, what is the hydrogen ion concentration?
A. 0.74 B. 0.008 C. 125.89 D. −0.322 E. −125.89

Questions 57-58:

57. Which of the following is an acceptable first step to solve the equation $\ln 2x^3 = 5$?

A. $3\ln 2x = 5$ B. $2x^3 = e \cdot 5$ C. $2x^3 = \dfrac{5}{\ln}$ D. $2x^3 = e^5$ E. $\ln 2x^3 = \ln 5$

58. What is the correct **exact** solution to the equation $\ln 2x^3 = 5$?

A. $\sqrt[3]{\dfrac{5}{2\ln}}$ B. $\sqrt[3]{\dfrac{5e}{2}}$ C. $\sqrt[3]{\dfrac{e^5}{2}}$ D. $\sqrt[3]{\dfrac{5}{2}}$ E. $\frac{1}{2}e^{5/3}$

Questions 59-60:

59. To solve the equation $20 = 3e^x + 5$, which of the following is an acceptable first step?

A. $4 = 3e^x$ B. $\ln 20 = x\ln(3\cdot e) + \ln 5$ C. $\ln 20 = x\ln(3) + \ln 5$ D. $15 = 3e^x$ E. $25 = 3e^x$

60. Report the correct **exact** solution to the equation $20 = 3e^x + 5$.

A. $\frac{\ln 4}{\ln 3}$ B. $\ln(4/3)$ C. $\frac{\ln 25}{\ln 3}$ D. $\ln(25/3)$ E. $\ln(5)$

61. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded quarterly. Find the amount in year t.

A. $2200(1.382)^{4t}$ B. $2200(1+\frac{3.82}{4})^{4t}$ C. $2200(1+\frac{0.0382}{4})^{4t}$ D. $2200(1+\frac{3.82}{4})^{t}$ E. $2200(1+\frac{0.382}{4})^{4t}$

62. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded continuously. Find the amount in year t.

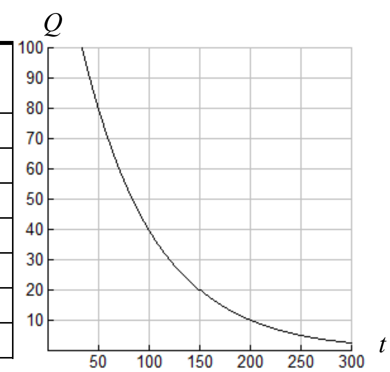
A. $2200e^{1.382t}$ B. $2200e^{1.0382t}$ C. $2200(e\cdot 1.382)^t$ D. $2200e^{0.382t}$ E. None of these

63. In year $t = 0$, the balance of an account is \$1000. The account earns 5% annual interest, compounded annually.
a) Find the amount in year 7, reported to the nearest \$0.01
b) By what total percent does the account increase by the end of the 7 year period?
Report to the nearest 0.01%.
c) Find the doubling time of the account, reported to the nearest 0.1 years.

64. The amount Q, in mg, of substance at year t decays according to the formula $Q = f(t)$, given in the table and graph. Assume the pattern holds and $f(t)$ is exponential.

a) Complete the first entry in the table.
b) Complete the next three rows of the table.
c) Report the half-life in years.
d) Find a formula for $Q = f(t)$
e) How many years will it take before the amount present first falls below 1 mg?
Report your answer to the nearest year.
f) Every year the amount decays by what percent?
Report your answer to the nearest 0.1%.

t, years	$Q = f(t)$, mg
0	
50	80
100	40
150	20
200	
250	
300	



65. Given $f(x) = \frac{4}{x^2}$ and $g(x) = \sqrt{x^2+4}$, find $f(g(x))$ and simplify.

A. $f(g(x)) = \frac{4}{x^2+4}$ B. $f(g(x)) = \frac{4}{\sqrt{x^2+4}}$ C. $f(g(x)) = x^2+4$ D. $f(g(x)) = \frac{4}{x^2\sqrt{x^2+4}}$ E. $f(g(x)) = \frac{1}{x^2}$

66. Given $f(x) = \frac{\sqrt{x+1}}{2}$ and $g(x) = x^2+3$, find $f(g(x))$ and simplify.

A. $f(g(x)) = \frac{x+2}{2}$ B. $f(g(x)) = \frac{\sqrt{x^2+4}}{2}$ C. $f(g(x)) = x$ D. $f(g(x)) = x+1$ E. $f(g(x)) = \frac{(x^2+3)\sqrt{x+1}}{2}$

67. Euphemia is solving the equation $(x+1)(x+2)=6$
She completes the problem as shown in the steps below.

Original Equation:	$(x+1)(x+2)=6$	
Step 1	$x+1=2$ and $x+2=3$	Since $2\cdot 3=6$, set the factors equal to 2 and 3, respectively.
Step 2	$x=1$	Solve each linear factor.

First determine if $x=1$ is the complete solution to the original equation $(x+1)(x+2)=6$
Then determine if there was an error made in the solution process.

A. Unfortunately, $x=1$ **is not** the complete solution to $(x+1)(x+2)=6$.
Her mistake is in **Step 1**.

B. Unfortunately, $x=1$ **is not** the complete solution to $(x+1)(x+2)=6$.
Her mistake is in **Step 2**.

C. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
She did not make any mistakes in the solution process.

D. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
However, her mistake is in **Step 1**.

E. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
However, her mistake is in **Step 2**.

68. Plutarch is solving the equation $(x+2)(x-2)=4$
He completes the problem as shown in the steps below.

Original Equation:	$(x+2)(x-2)=4$	
Step 1	$x+2=2$ and $x-2=2$	Since $2\cdot 2=4$, set the factors equal to 2 and 2, respectively.
Step 2	$x=0$ and $x=4$	Solve each linear factor.

First determine if $x=0$ and $x=4$ is the complete solution to the original equation $(x+2)(x-2)=4$.
Then determine if there was an error made in the solution process.

A. Unfortunately, $x=0$ and $x=4$ **is not** the complete solution to $(x+2)(x-2)=4$.
His mistake is in **Step 1**.

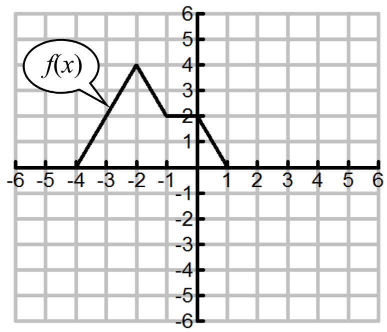
B. Unfortunately, $x=0$ and $x=4$ **is not** the complete solution to $(x+2)(x-2)=4$.
His mistake is in **Step 2**.

C. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
He did not make any mistakes in the solution process.

D. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
However, his mistake is in **Step 1**.

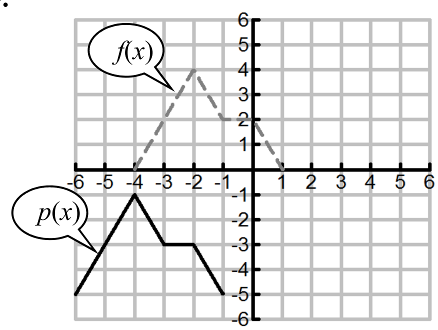
E. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
However, his mistake is in **Step 2**.

69-71. The graph of $y = f(x)$ is shown.

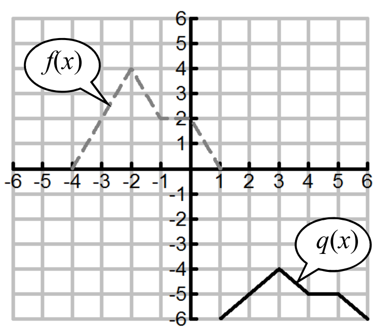


The functions shown below are transformations of $f(x)$. Write a rule for each function in terms of $f(x)$.

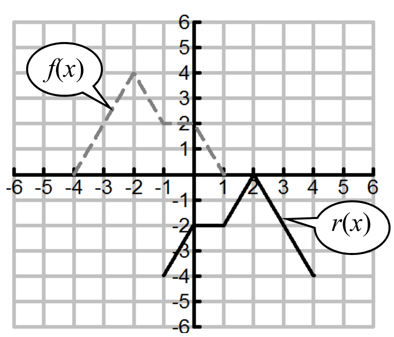
69.



70.

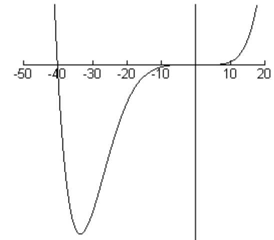


71.



Worked out solutions to all of these problems are posted on your instructor's Blackboard Site or at **www.pfw.edu/departments/coas/depts/math/courses/math-153.html** (The MA 15300 Course Website).

Solutions to Review for the MA 153 Final



$-50 \le x \le 20$

$-300{,}000{,}000 \le y \le 100{,}000{,}000$

1. For positive or negative large values of x,

$$f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$$

looks like its leading term, the power function $y = x^6$.
We can describe its long run behavior as follows:
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$.
Enlarge the viewing window to see that eventually the graph turns around.
Choice **B**.

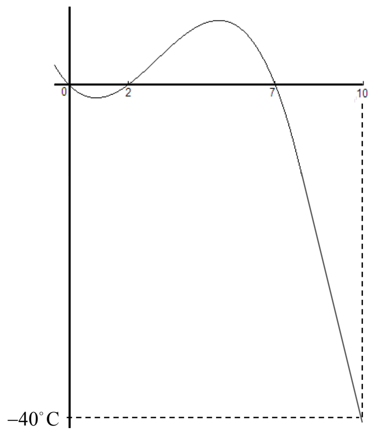
2. There are zeros at 0, 2, and 7. Therefore: $y = kt(t-2)(t-7)$

$$x = 1,\ y = -1) \Rightarrow\ -1 = k(1)(1-2)(1-7)$$
$$-1 = k(-1)(-6)$$
$$k = -\tfrac{1}{6}$$

The minimum value of $P(t)$ in the first ten seconds must be $P(10) = -40^\circ\text{C}$. This can be found using a graph or table or by evaluating $P(t) = -\frac{1}{6}t(t-2)(t-7)$ for $t = 10$.

$P(10) = -\frac{1}{6}(10)(10-2)(10-7) = -\frac{1}{6}(10)(8)(3) = -40$.

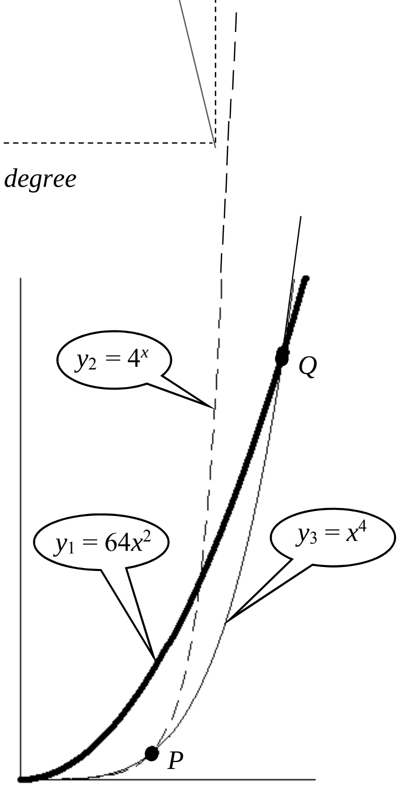
Choice **D**.



3. $Q(t) = -\frac{1}{6}t^3$ since $P(t) = -\frac{1}{6}t(t-2)(t-7) = -\frac{1}{6}t^3 +$ *remaining terms of lower degree*

Therefore, $P(t)$ and $Q(t)$ look very much alike for large values of t.

(Note that the $-\frac{1}{6}$ is not optional.)



4. (a) Since all global behavior is shown, notice in the long run that graph y_2 is above graph y_3 which is above graph y_1. Exponential functions eventually outpace power functions, so the graph of $y = 4^x$ will be above the graphs of $y = 64x^2$ and $y = x^4$. Power functions with greater degree will outpace those of lower degree, so the graph of $y = x^4$ must eventually be above the graph of $y = 64x^2$.
Since $y_1 = 64x^2$, $y_2 = 4^x$, and $y_3 = x^4$ is not one of the choices, the correct answer to part **(a)** is **E**. None of these.

(b) The point P is where the graphs of $y_2 = 4^x$ and $y_3 = x^4$ intersect.
We must solve the equation $4^x = x^4$.
This equation can not be solved with logarithms, however, since the coordinates of the intersection points are integers, enter both equations into a graphing calculator and use a table feature. They intersect at (2, 16) and (4, 256). The graph shows that $y_2 = 4^x$ and $y_3 = x^4$ do not intersect after P, so the P is the point (4, 256).

	4^x	x^4
X	Y2	Y3
0	1	0
1	4	1
2	16	16
3	64	81
4	256	256
5	1024	625
6	4096	1296
7	16384	2401
8	65536	4096

The point Q is where the graphs of $y_1 = 64x^2$ and $y_3 = x^4$ intersect. We must solve the equation $64x^2 = x^4$, which we can do analytically by factoring:

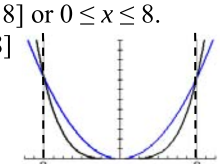
$$64x^2 - x^4 = 0$$
$$x^2(64 - x^2) = 0$$
$$x^2 = 0 \quad\Big\|\quad x^2 = 64$$
$$x = 0 \quad\Big\|\quad x = \pm 8$$

	$64x^2$	x^4
X	Y1	Y3
0	0	0
1	64	1
2	256	16
3	576	81
4	1024	256
5	1600	625
6	2304	1296
7	3136	2401
8	4096	4096
9	5184	6561
10	6400	10000

Use a table or substitution to find the value of y if $x = 8$, so Q is the point (8, 4096).

(c) For nonnegative values of x, the graph of $y_1 = 64x^2$ intersects the graph of $y_3 = x^4$ at $x = 0$ and $x = 8$. In the first quadrant, the graph of $y_1 = 64x^2$ is also *above* the graph of $y_3 = x^4$ from $x = 0$ and $x = 8$. Therefore for $x \ge 0$ the values of x which solve $y_1 \ge y_3$ are those in the interval [0, 8] or $0 \le x \le 8$.

(d) Both $y_1 = 64x^2$ and $y_3 = x^4$ are symmetric about the y-axis (even), so the solution is $[-8, 8]$ or $-8 \le x \le 8$. TIP: If solving graphically, use the table to find a suitable window such as $-10 \le x \le 10$ and $0 \le y \le 6000$.



5. $E(t) = 30t^{0.668}$. To find $y = kt^p$, notice $E(1) = 30$ so if $t = 1$, then $y = 30$.

Therefore we have $k = 30$, since $30 = k(1)^p = k(1) = k$.

Now use another point to find p for $y = 30t^p$. We used (2.02, 48).

$48 = 30(2.02)^p$

$\frac{48}{30} = (2.02)^p$

$1.6 = (2.02)^p$ So $p = \frac{\ln 1.6}{\ln 2.02} \approx 0.668.$

This means $E(t) = 30x^{0.67}$ and $E(7) = 30(7)^{0.67} \approx 110.$ Choice **B**.

6. $S(t) = 5.61x^{1.37}$. To find $y = kt^p$, use two points. We used (2.05, 15) and (2.98, 25).

$\frac{25}{15} = \frac{k(2.98)^p}{k(2.05)^p}$

$\frac{5}{3} = (\tfrac{2.98}{2.05})^p$

$p = \frac{\ln(5/3)}{\ln(2.98/2.05)} \approx 1.3655$

$y = kt^{1.366}$ Now use any other point to find k. We used (1.05, 6)

$6 = k(1.05)^{1.366}$

$k \approx 5.61$

$S(t) = 5.61x^{1.37}$ Choice **E**.

7. $E(t) = 30x^{0.67}$

$S(t) = 5.61x^{1.37}$

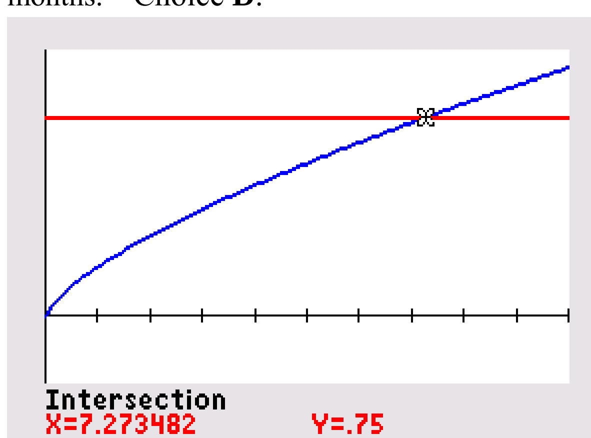
$\frac{S}{E} = \frac{5.61x^{1.37}}{30x^{0.67}} = 0.187x^{0.7}$

Solve $0.187x^{0.7} > 0.75$ by graphing $y = 0.187x^{0.7}$ and the target line $y = 0.75$

Perform an INTERSECTION routine or solve $0.187x^{0.7} = 0.75$ to find the first time after which the ratio $\frac{S}{E}$ is above 0.75. This is about 7.3 months. Choice **D**.

```
Y1=.187X^.7
Y2=.75
```

Window: $0 \le x \le 10,\ -0.25 \le y \le 1$



Note: You could also just enter

```
Y1=5.61X^1.37/(30X^.67)
Y2=.75
```

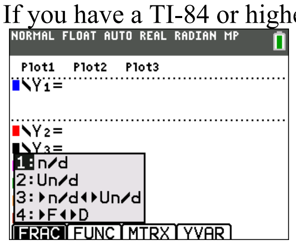
in a grapher, but the parentheses are crucial on a TI-83 or TI-83 Plus.

For example, you would NOT get the same function if you just typed

```
Y1=5.61X^1.37/30X^.67
Y2=.75
```

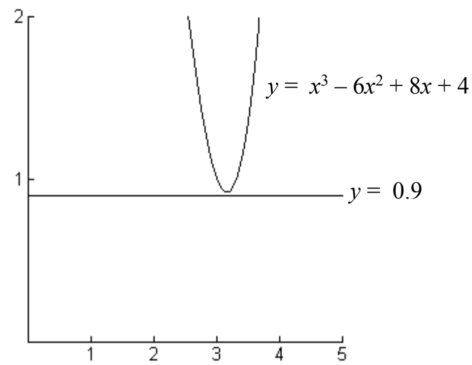
That would give you $y = \dfrac{5.61x^{1.37}}{30} \cdot x^{0.67}$ which is not what you want at all.

If you have a TI-84 or higher, use the fraction template **n/d** by pressing ALPHA Y=.



$$Y_1 = \frac{5.61X^{1.37}}{30X^{.67}}$$
$$Y_2 = .75$$

8. The town of *Polynomia* always exceeds 90 people.
A population of 90 people = 0.9 hundred.
Use a graphing calculator to sketch
$y = x^3 - 6x^2 + 8x + 4$ and the line $y = 0.9$
in a viewing window such as
$0 \le x \le 5$ and $0 \le y \le 2$
The polynomial never falls below the line.
You could also use a calculator routine to determine the minimum value of the polynomial in this window, which is (3.1547, 0.920799).
For $t > 0$ the lowest value of this town's population is a mere 92 people!



$P(t) = t^3 - 6t^2 + 8t + 4$ has the same long run behavior as $y = x^3$, so it will continually increase after $t = 4$ or after 1974. Since the town of *Exponentia* begins initially with 400 people and grows by 20% each year, the formula for $E(t) = 4(1.2)^t$. The graph of this function increases for all t. Exponential functions will eventually outpace polynomial functions, so the graphs must cross more than three times. Using graphing technology, we can find that $E(t)$ will intersect $P(t)$ again about 58.88 years after 1970, or in the year 2028. To check this, sketch the difference function $D(t) = E(t) - P(t)$ on a grapher and find when it is zero.

The correct response is Choice **E**, all of the above are true.

9. We have $C(t) = \dfrac{P(t)}{R(t)} = \dfrac{360 + 9t}{12{,}000 + 12t}$. Therefore $C(0) = \dfrac{360 + 9(0)}{12{,}000 + 12(0)} = \dfrac{360}{12{,}000} = 0.03$ or 3%. Choice **B**.

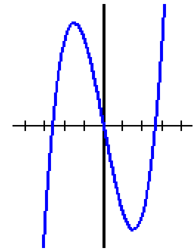
10. As t gets larger and larger, the function $C(t) = \dfrac{360 + 9t}{12{,}000 + 12t}$ approaches the ratio of the leading terms, namely $\dfrac{9t}{12t} = 0.75$. Eventually 75% of the reservoir's total volume would consist of pollutants. This can be confirmed with a graph of the function or a view of its table for large values of t. Choice **E**.

11. a) The zeros of $f(x)=400x(6x^2-42)$ are 0, $\sqrt{7}$, and $-\sqrt{7}$

Check graphically.

This third degree polynomial crosses the x-axis three times.

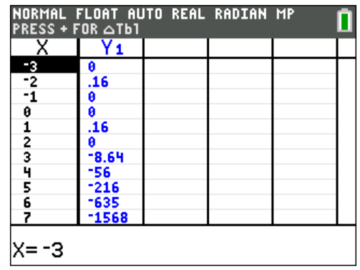
Find the zeros by solving $f(x) = 0$. Set each factor equal to 0 and solve.



$$400x(6x^2-42)=0$$

$400x=0$	$6x^2-42=0$
$x=0$	$6x^2=42$
	$x^2=7$
	$x=\pm\sqrt{7}$

b) The zeros of $f(x)=-\frac{1}{100}(x-2)(x^3+3x^2)(x+1)^2$ are 2, 0, −3, −1.


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y_1
-3	0
-2	.16
-1	0
0	0
1	.16
2	0
3	-8.64
4	-56
5	-216
6	-635
7	-1568

X=-3

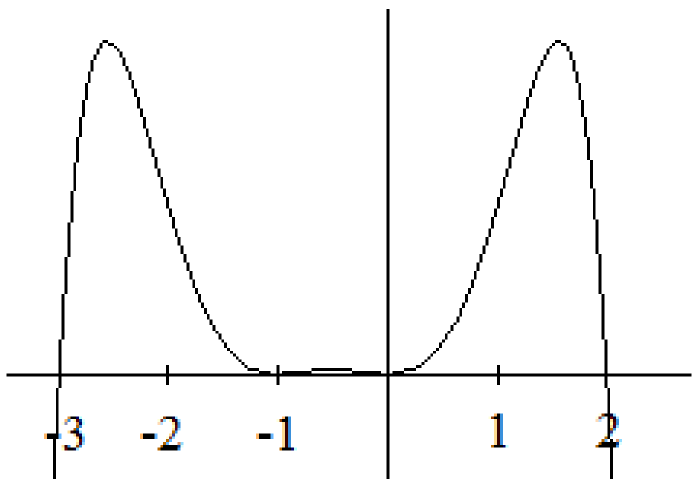
Set $f(x) = 0$ and factor completely:

$$-\tfrac{1}{100}(x-2)(x^3+3x^2)(x+1)^2=0$$

$$-\tfrac{1}{100}(x-2)x^2(x+3)(x+1)^2=0$$

$-\frac{1}{100}(x-2)=0$	$x^2=0$	$x+3=0$	$(x+1)^2=0$
$x=2$	$x=0$	$x=-3$	$x=-1$

Notice the graph crosses the x-axis four times, with double zeros at −1 and 0 (where it bounces) and single zeros at −3 and 2..



c) $f(x)=10x^3-4x^4-4x^2$ has zeros at 0, $\frac{1}{2}$, and 2.

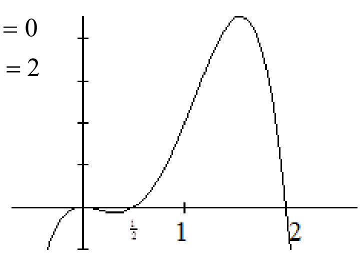
$$10x^3-4x^4-4x^2=0$$

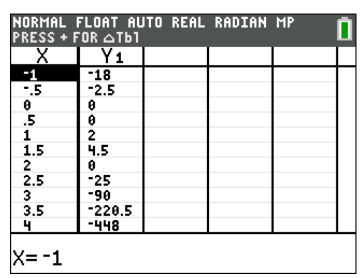
$-4x^4+10x^3-4x^2=0$ Arrange in descending powers of *x*.

$-2x^2\left(2x^2-5x+2\right)=0$ Remove the greatest common factor.
To factor a quadratic trinomial, make its leading term positive.

$-2x^2\left(2x-1\right)(x-2)=0$ Viewing a table or graph can help you factor.

$-2x^2=0$	$2x-1=0$	$x-2=0$
$x=0$	$x=\frac{1}{2}$	$x=2$




NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y_1
-1	-18
-.5	-2.5
0	0
.5	0
1	2
1.5	4.5
2	0
2.5	-25
3	-90
3.5	-220.5
4	-448

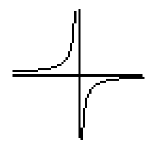
X=-1

12. Report the leading term, the leading coefficient, and the degree for each polynomial.

Polynomial	leading term	leading coefficient	degree
a) $f(x) = 400x(6x^2 - 42)$ $= 2400x^3 -$ (**a term of lower degree**)	$2400x^3$	2400	2
b) $f(x) = -\frac{1}{100}(x-2)(x^3+3x^2)(x+1)^2$ $= -\frac{1}{100}x^6 +$ (**terms of lower degree**)	$-\frac{1}{100}x^6$	$-\frac{1}{100}$	6
c) $f(x) = 10x^3 - 4x^4 - 4x^2$	$-4x^4$	-4	4

The graphs of the leading terms share the same long run behavior as the graphs of f(x) that are shown in the previous Question 11.

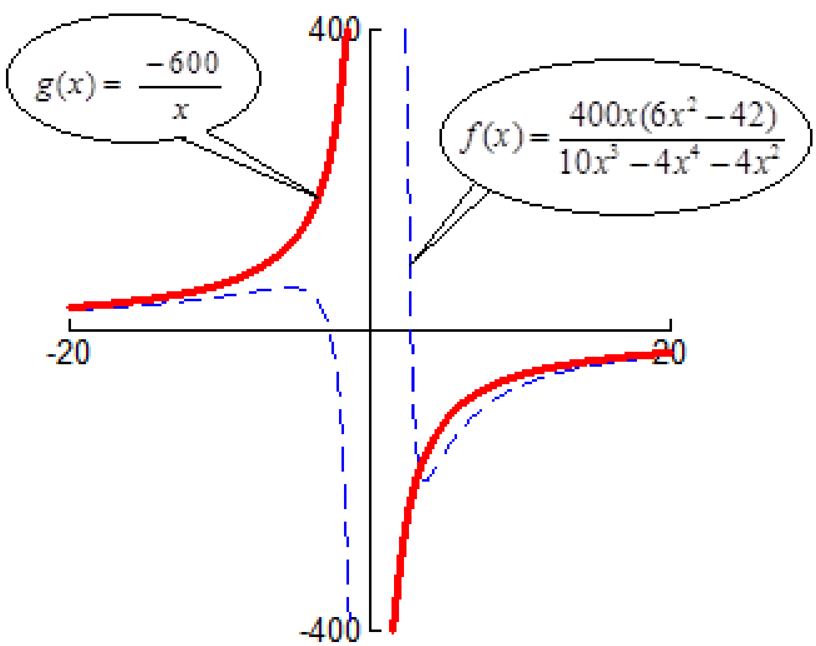
13. Report the formula of the power function *g*(*x*) which has the same long run behavior as *f*(*x*), graph *g*(*x*), and, if it exists, report the equation of the horizontal asymptote of *f*(*x*).

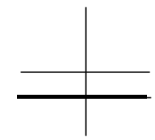
Rational Function	*g*(*x*)	Graph of *g*(*x*)	Horizontal Asymptote
a) $f(x) = \dfrac{400x(6x^2-42)}{10x^3-4x^4-4x^2}$ $\approx \dfrac{2400x^3}{-4x^4} = \dfrac{-600}{x} \to 0 \;\; as\, x \to \pm\infty$	$g(x) = \dfrac{-600}{x}$		$y = 0$

If you enter *f*(*x*) and *g*(*x*) into a grapher, you could compare their tables and graphs for large values of *x*. Using the formula, however, can be much faster. Creating the table and the graph is not necessary to solve the problem.

Y1 = 400X(6X²-42)/(10X³-4X⁴-4X²)
Y2 = -600/X

X	Y1	Y2
-25000	.024	.024
-20000	.03	.03
-15000	.03999	.04
-10000	.05998	.06
-5000	.11994	.12
0	ERROR	ERROR
5000	-.1201	-.12
10000	-.06	-.06
15000	-.04	-.04
20000	-.03	-.03
25000	-.024	-.024



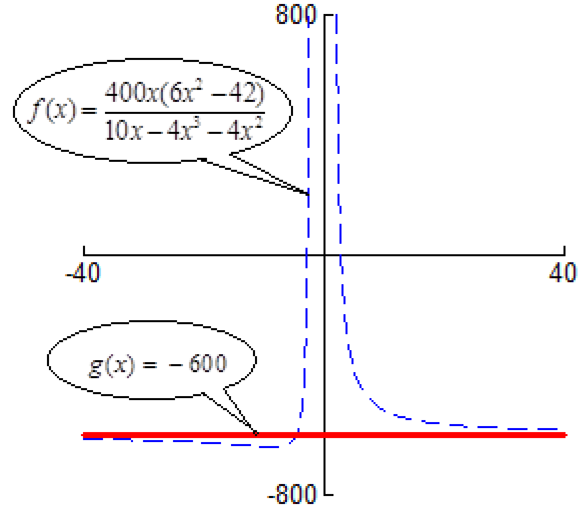
Rational Function	$g(x)$	Graph of $g(x)$	Horizontal Asymptote
b) $f(x)=\dfrac{400x(6x^2-42)}{10x-4x^3-4x^2} \approx \dfrac{2400x^3}{-4x^3} = -600 \quad as\, x \to \pm\infty$	$g(x) = -600$		$y=-600$

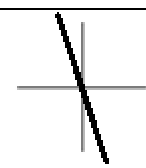
As before, creating the table and the graph is not necessary to solve the problem, but they are provided to show that the function $f(x)$ looks like $g(x)$ for large values of x.

$Y_1 = \dfrac{400X(6X^2-42)}{10X-4X^3-4X^2}$

$Y_2 = {}^-600$

X	Y1	Y2
-25000	-600	-600
-20000	-600	-600
-15000	-600	-600
-10000	-600.1	-600
-5000	-600.1	-600
0	ERROR	-600
5000	-599.9	-600
10000	-599.9	-600
15000	-600	-600
20000	-600	-600
25000	-600	-600



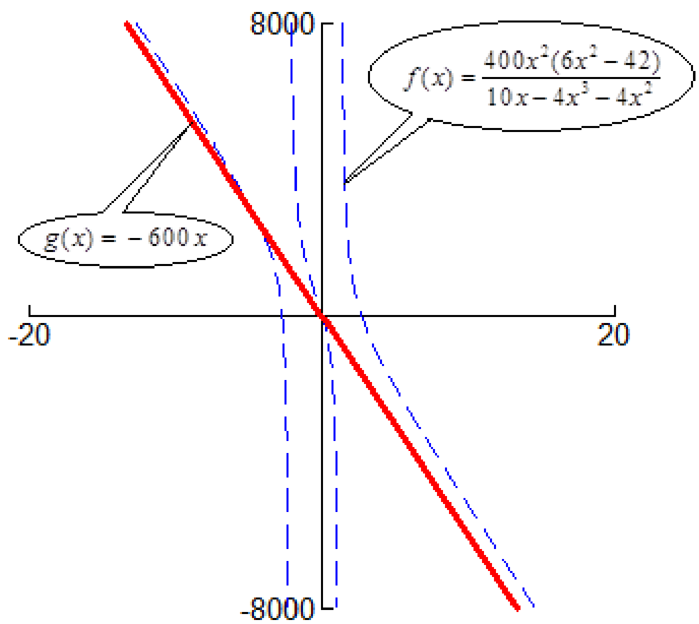
Rational Function	$g(x)$	Graph of $g(x)$	Horizontal Asymptote
c) $f(x)=\dfrac{400x^2(6x^2-42)}{10x-4x^3-4x^2} \approx \dfrac{2400x^4}{-4x^3} = -600x \quad as\, x \to \pm\infty$	$g(x) = -600x$		None

The table and the graph are provided to show that the function $f(x)$ looks like $g(x)$ for large values of x.

$Y_1 = \dfrac{400X^2(6X^2-42)}{10X-4X^3-4X^2}$

$Y_2 = {}^-600X$

X	Y1	Y2
-25000	1.5E7	1.5E7
-20000	1.2E7	1.2E7
-15000	9E6	9E6
-10000	6E6	6E6
-5000	3E6	3E6
0	ERROR	0
5000	-3E6	-3E6
10000	-6E6	-6E6
15000	-9E6	-9E6
20000	-1.2E7	-1.2E7
25000	-1.5E7	-1.5E7



14. 30 lb of fertilizer produces a maximum yield of 400 pecks of peppers. Choice **B**.

15. Without applying any fertilizer at all, we see from the graph that the orchard will produce 175 pecks of peppers. Choice **C**.

16. The range is $0 \le f(m) \le 400$. Note: You can also write [0, 400]. Choice **E**.

17. The function $f(m)$ is **increasing** for $0 < m < 30$. Choice **C**.
Note: The function $f(m)$ is **decreasing** for $30 < m < 70$.

18. The function $f(m)$ is never **concave up** and is **concave down** for $0 < m < 70$. Choice **E**.

19. $f(m) > 175$ for $0 < m < 60$.
Determine where the graph of $y = f(m)$ is above the line $y = 175$.
The yield is more than 175 pecks of peppers when the amount of fertilizer applied is more than 0 lb and less than 60 lb. Choice **D**.

20. **a)** The equation of the axis of symmetry is the line $x = 30$. (Note: reporting just the number 30 is incorrect.) You could also report the equation of the line as $m = 30$.

b) To find the vertex form, use a shift transformation of the graph of $y = ax^2$ (left 30 and up 400). We have $y = a(x - 30)^2 + 400$. Plug in the point (0, 175).

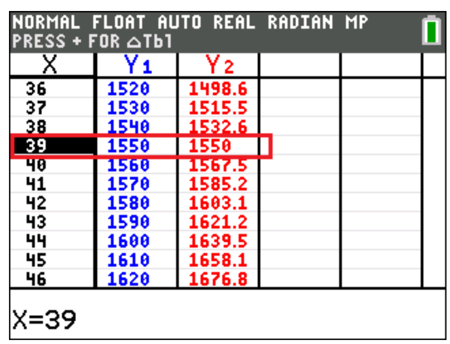
$$y = a(x-30)^2 + 400$$
$$175 = a(0-30)^2 + 400$$
$$-225 = 900a$$
$$a = -0.25$$

In vertex form, $f(x) = -0.25(x - 30)^2 + 400$.

21. The function has a positive zero of 70, which is $70 - 30 = 40$ units from the axis of symmetry.
The other zero is also 40 units from the axis of symmetry or at $30 - 40 = -10$.
The factored form is $y = a(x - 30)(x + 10)$, but from Question 20b, $a = -0.25$.
In factored form, $f(x) = -0.25(x - 70)(x + 10)$.

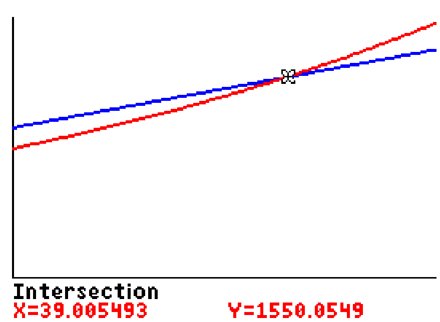
Note: if we had not solved for a in Question 20b, you can also find a by plugging in the point (0, 175). However, the leading coefficient a is the same for expanded form, vertex form, and factored form.

22. $P = 1160 + 10t$ and $Q = 1000(1.0113)^t$
Set the equations equal to each other and solve using technology.
They intersect at $t = 39$ years. Choice **C**.


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1	Y2
36	1520	1498.6
37	1530	1515.5
38	1540	1532.6
39	1550	1550
40	1560	1567.5
41	1570	1585.2
42	1580	1603.1
43	1590	1621.2
44	1600	1639.5
45	1610	1658.1
46	1620	1676.8

X=39



23. $Q = 20(0.4)^t = 20(1 - 0.6)^t$, so 60% of the drug is lost per hour. Choice **E**.

24. The growth factor of $y = ab^t$ is b. Choice **A**.

25. The equation is $P = 9216(1.125)^t$. The initial amount when $t = 0$ is \$9,216. Choice **C**.

$$\frac{ab^{18}}{ab^{3}} = \frac{76787.03}{13122}$$

$$\frac{\cancel{a}b^{18}}{\cancel{a}b^{3}} = \frac{76787.03}{13122}$$

$$b^{15} = \frac{76787.03}{13122}$$

$$b = \sqrt[15]{\frac{76787.03}{13122}} = \left(\frac{76787.03}{13122}\right)^{\frac{1}{15}} = 1.125$$

$$y = a(1.125)^t$$

$$13122 = a(1.125)^3$$

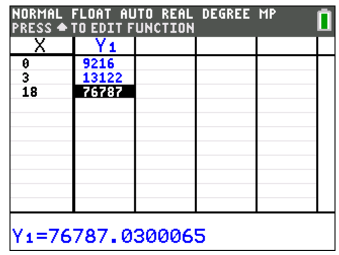
$$a = \frac{13122}{(1.125)^3} = 9216$$

$$P = 9216(1.125)^t$$

Y1=9216(1.125)^X

Press 2nd TBLSET and use the **Indpnt: Auto** feature to be able to enter nonconsecutive inputs.

The table confirms the equation is true. (Slight roundoff error is of no concern.)



26. Since the equation is $P = 9216(1.125)^t = 9216(1+ \underline{\mathbf{0.125}})^t$, the growth rate is 12.5%. Choice **C**.

27. (a) We have been given that the equation is of the form $y = ab^t + 60$ and we must find a and b.

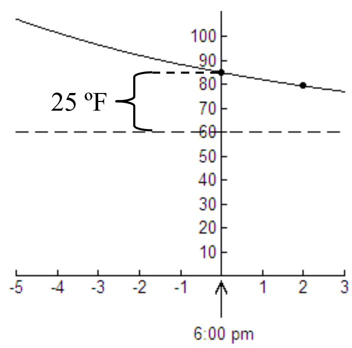
When $t = 0$, $y = 85$ °F:

$85 = ab^0 + 60$

$85 = a + 60$

$a = 85 - 60 = 25$.

Note this is the initial temperature difference between the butler and the room temperature.



We have $y = 25b^t + 60$ and need b.

When $t = 2$, $y = 79.36$ °F:

$79.36 = 25b^2 + 60$

$19.36 = 25b^2$

$$b^2 = \frac{19.36}{25} = 0.7744$$

$$b = \sqrt{0.7744} = 0.88$$

The model is $y = 25(0.88)^t + 60$. Check with a grapher or resubstitute the points. Choice **I.**

(b) We must write $y = 25(0.88)^t + 60$ as $y = He^{kt} + 60$.

We can simplify this to writing $25(0.88)^t$ as He^{kt} for some constants H and k.

The constant H is 25.

To find k, set $e^k = 0.88$

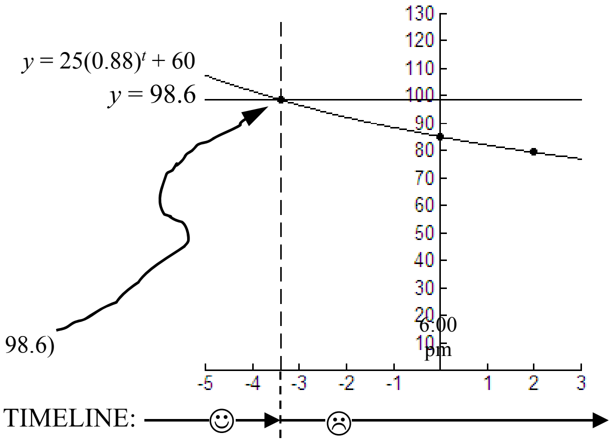
so $k = \ln e^k = \ln(0.88)$

To 3 decimal places, $k = \ln(0.88) = -0.128$ and we have $y = 25e^{-0.128t} + 60$.

Again we can check with a grapher or resubstitute the points. Choice **I.**

(c) When his body temperature, y, is 98.6ºF, we will assume the butler was alive.
Set $y = 25(0.88)^t + 60$ and $y = 98.6$ equal to each other to find the time of death.
(From the graph, we expect a negative number.)
Algebraic solution:

$$25(0.88)^t + 60 = 98.6$$
$$25(0.88)^t = 38.6$$
$$(0.88)^t = \tfrac{38.6}{25}$$
$$\ln(0.88)^t = \ln\tfrac{38.6}{25}$$
$$t\ln(0.88) = \ln(38.6/25)$$
$$t = \tfrac{\ln(38.6/25)}{\ln(0.88)} \approx -3.4 \quad (-3.4, 98.6)$$



Notice the timeline on the graph:
He died 3.4 hours before 6:00 pm.

<Before death, butler is 98.6ºF >< Butler starts to cool.>

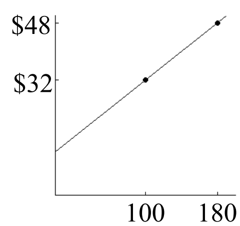
This is 3 hours and $0.4 \cdot 60 = 24$ minutes before 6:00 pm
(or 24 minutes prior to 3:00 pm) which is 2:36 pm. Since the house records indicate that the niece arrived at 2:45 pm., the butler was already dead when she arrived.

Note: You could have also have used the equation involving e as shown below.
Since $\ln(0.88) = -0.128$, you reach the same answer:

$$25e^{-0.128t} + 60 = 98.6$$
$$25e^{-0.128t} = 38.6$$
$$e^{-0.128t} = \tfrac{38.6}{25}$$
$$\ln e^{-0.128t} = \ln\tfrac{38.6}{25}$$
$$-0.128t = \ln(38.6/25)$$
$$t = \tfrac{\ln(38.6/25)}{-0.128} \approx -3.4$$

28. (a) It might be helpful to plot the points and organize the information in a table.

w	C
100	\$32
180	\$48



The slope is positive:

$$m = \frac{\Delta C}{\Delta w} = \frac{\$48 - \$32}{180 - 100} = \frac{16}{80} = 0.2$$

We have $C = b + 0.2w$
Substitute $w = 100$, $C = \$32$:

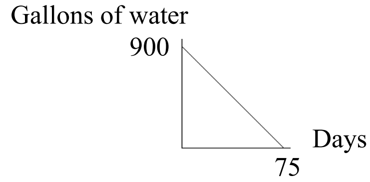
$$32 = b + 0.2(100)$$
$$32 = b + 20$$
$$b = 12$$

Therefore $C = 12 + 0.2w$.

(b) The slope is \$0.20 per kg, which is the monthly rate that the service charges for waste collection.

(c) The vertical intercept is (0, \$12). When no waste is collected, the service charges a fixed charge of \$12.

29. (a) Since we start with 900 gallons of fresh water, the vertical intercept is (0, 900). Each day we lose 12 gallons of water so the equation is $f(t) = 900 - 12t$.



(b) (i) $f(0) = 900$.
Initially we have 900 gallons of water.

(ii) To find $f^{-1}(0) = t$, we must find the time t when the team has 0 gallons of fresh water.

$0 = 900 - 12t$

$12t = 900$

$t = \frac{900}{12} = 75$

Thus $f^{-1}(0) = 75$ days.

It will take 75 days before the team has 0 gallons of water remaining.

30. (a) When we have zero U.S. dollars, we have zero shillings: the y-intercept is (0, 0).

(b) We need an equation for $y = f(x)$.
We first find the slope of the function.
The function is increasing so we expect a positive slope.
One way is to find the slope is to compute Δy and Δx.

Δx	U.S. dollars	Shillings	Δy
	\$1.00	2200	
\$1.50	\$2.50	5500	3300
\$0.50	\$3.00	6600	1100

We want $\frac{\Delta y}{\Delta x} = \frac{3300 \text{ shillings}}{\$1.50} = 2200$

Check that this is also the same as $\frac{1100 \text{ shillings}}{\$0.50} = 2200$ shillings per U.S. dollar.

Since the y-intercept is (0, 0), the equation is $y = 2200x$.
Check: The equation passes through the point (1, 2200), as well as the other points in the table.
If $y = 4000$ shillings, then $4000 = 2200x$ so $x = \frac{4000}{2200} \approx \$1.82$. The trousers cost \$1.82.
(Recall these are second-hand items in the Kampala market.)

(c) If we have $x = \$4.00$, then we can exchange it for $y = 2200x = 2200 \cdot 4 = 8800$ shillings, so we can afford the 8500 shilling coat.

31. (a) We know that when the price $p = \$10$, the number of customers N who will come to the park is 10,000. For each \$1.00 increase in the entrance price p, the park would lose an average of 500 daily customers:

p	N
\$10.00	10,000
\$11.00	9,500
\$12.00	9,000
\$13.00	8,500
\$14.00	8,000

(b) $N = f(p)$ is linear. When $\Delta p = \$1$, then $\Delta N = -500$.

The slope is $\frac{\Delta N}{\Delta p} = \frac{-500}{\$1} = -500$ and it passes through (\$10, 10,000).

We have $N = b - 500p$. Substitute $p = 10$, $N = 10{,}000$:

$10{,}000 = b - 500(10)$

$10{,}000 = b - 5{,}000$

$b = 15{,}000$

Therefore $N = f(p) = 15{,}000 - 500p$. *TIP:* Check the formula using the table feature of a grapher.

(c) If 10,000 customers pay \$10 each, the revenue is $10{,}000 \cdot \$10 = \$100{,}000$. Do this for each row to complete the table. Notice revenue increases due to the ticket price increase, although N decreases

p	N	$R = p \cdot N$
\$10.00	10,000	\$100,000
\$11.00	9,500	\$104,500
\$12.00	9,000	\$108,000
\$13.00	8,500	\$110,500
\$14.00	8,000	\$112,000

(d) In general, R is the product of the first two columns, so $R = p \cdot N$. Since $N = 15{,}000 - 500p$, we have $R = p \cdot N = p \cdot (15{,}000 - 500p)$.
Check the formula using the table feature of a grapher

(e) To find the N-intercept of $N = 15{,}000 - 500p$, set $p = 0$. By inspection it is (0, 15,000).
Interpretation: If the tickets were free, the amusement park would have 15,000 customers.
To find the p-intercept of $N = f(p)$, set $N = 0$ and solve for p:

$$\begin{aligned} N &= 15000 - 500p \\ 0 &= 15000 - 500p \\ 500p &= 15000 \\ p &= 30 \end{aligned}$$

The p-intercept is (30, 0).
Interpretation: If the ticket price was \$30, no customer would purchase one.

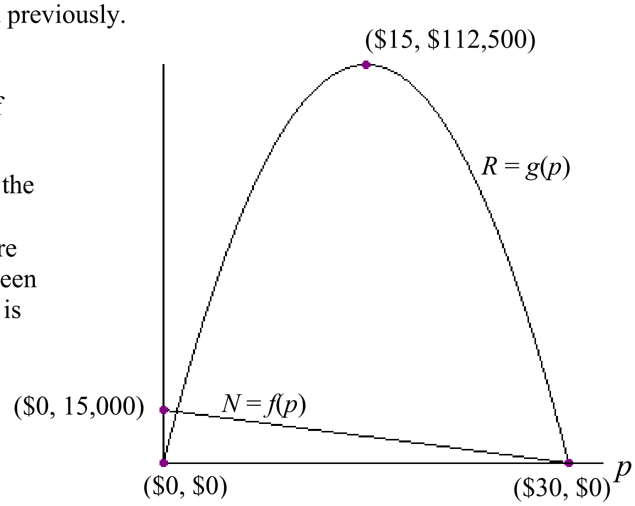
(f) Find any p-intercepts by solving $R = p \cdot (15{,}000 - 500p) = 0$
Set each factor equal to 0: we know $R = 0$ when $p = 0$ and $15{,}000 - 500p = 0$.
From part **(e)**, $15{,}000 - 500p = 0$ when $p = 30$ so the p-intercepts are (0, 0) and (30, 0).
Interpretation:
(0, 0): If the tickets were free,
there would be no revenue (even though 15000 customers would come).
(30, 0): If the tickets were \$30, there would be no revenue (since no customers would buy them.)

To find all the R-intercepts, set $p = 0$ in the equation $R = g(p) = p \cdot (15{,}000 - 500p)$.
$R = g(0) = 0 \cdot 15{,}000 = 0$.
The only R-intercept is the point (0, 0), interpreted previously.

(g) \$15 since the maximum is at (15, 112,500).
There are several strategies to get the maximum of
$R(x) = x(15{,}000 - 500x) = -500x^2 + 15000x$

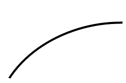
1. The coefficient of the x^2 term is negative, so the parabola is concave down. Since a parabola is symmetric about its maximum, and its zeros are at $x = 0$ and 30, the maximum is midway between at $x = 15$. The y= coordinate of the maximum is $R = g(15) = 15 \cdot (15{,}000 - 500 \cdot 15) = 112{,}500$.
2. Use the maximum feature.
3. Use the table feature.

(h) See the graph at the right.

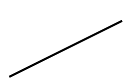


32. $e^{x \ln a} = e^{\ln a^x} = a^x$. Choice **E**.

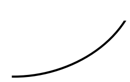
33. (a) Choice III



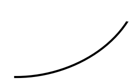
(b) Choice VI



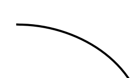
(c) Choice IV



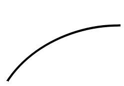
(d) Choice IV



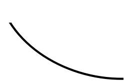
(e) Choice I



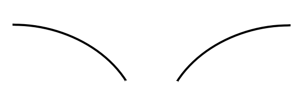
(f) Choice III



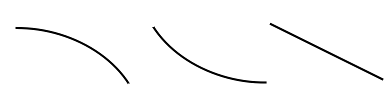
(g) Choice II



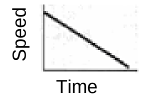
(h) Choices I and III



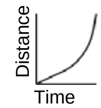
(i) Choices I, II, and V



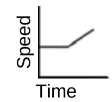
34. **a**. Choice II. The train's speed slows to a stop (speed is 0).



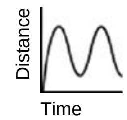
b. Choice I. My rate is constant at first, so the graph appears linear.
Once the chimes ring, my rate increases so the graph is concave up.



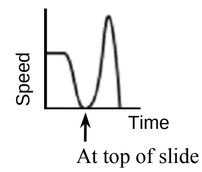
c. Choice III. First my speed is constant, or flat. The graph appears horizontal.
When I run, my speed increases.



d. Choice II. The ferris wheel car climbs to its highest point, then descends, then climbs again.



e. Choice III. As the child climbs up the slide her speed is steady and constant.
When she stops at the top of the slide, her speed is 0.
Once she slides down her speed increases, exceeding the speed she had when she was climbing the slide.
At the bottom of the slide, her speed is 0 when she stops.



35. **(i)** $P(t) = 300 - 2t$ is Choice **F** since $300 - 2t = 250$ when $t = 25$.
The population starts at 300 and has dropped to 250 after 25 years. It is not Choice **A** since, even though $P(t) = 300 - 2t$ declines at a constant rate, $P(t)$ becomes 0 in 150 years, not 15.

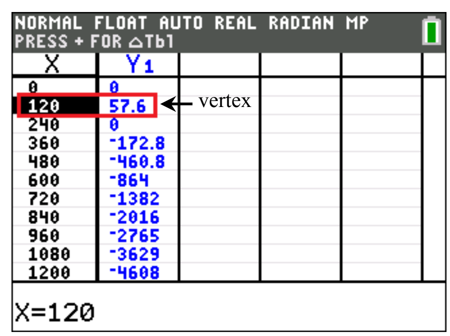
(ii) $Q(t) = 300e^{0.02t}$ is Choice **C**. *The population, which began at 300, is growing at the continuous rate of 2 percent each year.*

(iii) $R(t) = 300(0.98)^t$ is Choice **H**. *The population, originally at 300, has been decreasing at the annual rate of 2 percent.*

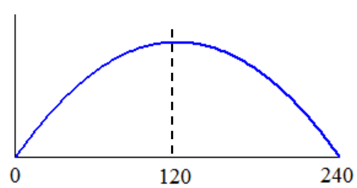
(iv) $S(t) = -\frac{4}{3}t^2 + 300$ is Choice **G**. The population, which began at 300, decreases faster and faster.

36. **a)** Factor $h(x)$ to find the zeros.

$$0.96x - 0.004x^2 = 0$$
$$x(0.96 - 0.004x) = 0$$
$$x = 0 \quad \Big\| \quad 0.96 - 0.004x = 0$$
$$0.96 = 0.004x$$
$$x = \frac{0.96}{0.004} = 240$$



$h(x)$ has zeros at 0 and 240 and is concave down since $a = -0.004$.
Use a table to find the vertex, which is halfway between the zeros at the point (120, 57.6) so the exact maximum height is 57.6 ft.



b) The shell hits the ground 240 feet from the base.
c) The vertex is (120, 57.6).
d) The equation of the axis of symmetry is $x = 120$. Note: reporting just 120 is not correct.
e) In vertex form $h(x) = a(x-120)^2 + 57.6$, but since $h(x) = -0.004x^2 + 0.96x$, the value of $a = -0.004$.
So $h(x) = -0.004(x-120)^2 + 57.6$
f) In factored form $h(x) = -0.004x(x-240)$. Note: Although $h(x) = x(0.96 - 0.004x)$ is also in factored form, the equation $h(x) = -0.004x(x-240)$ has the advantage of showing the zeros.

37. (a) The horizontal asymptote is $y = 4$. The vertical asymptotes are $x = 0$ and $x = 2$.

To find if there is a horizontal asymptote, examine the long run behavior:

$$y = \frac{8x^2 - 8}{2x^2 - 4x} \rightarrow \frac{8x^2}{2x^2} = 4 \quad \text{as} \quad x \rightarrow \pm\infty$$

Since the function looks like the line $y = 4$ for very large values of x, the line $y = 4$ is the horizontal asymptote.

To find the vertical asymptotes, factor the denominator:

$$y = \frac{8x^2 - 8}{2x^2 - 4x} = \frac{8(x^2 - 1)}{2x(x-2)} = \frac{4(x-1)(x+1)}{x(x-2)}$$

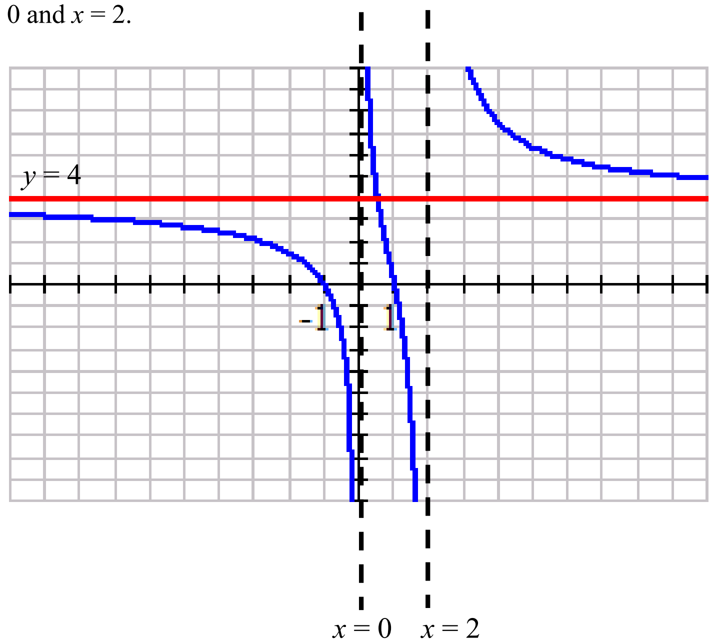
The function has vertical asymptotes when the denominator is zero (and the numerator is not).

The denominator $x(x-2) = 0$ when $x = 0$ and $x = 2$.

Y1=(8X²-8)/(2X²-4X)
Y2=4

X	Y1
-3	2.1333
-2	1.5
-1	0
0	ERROR
1	0
2	ERROR
3	10.667
4	7.5
5	6.4
6	5.8333
7	5.4857

There is a vertical asymptote at $x = 0$ and $x = 2$.



(b) We can find the domain of $f(x) = \sqrt{x-100}$ using the graph, the table or reason from the formula.

The graph of $f(x) = \sqrt{x-100}$ is a horizontal shift of the graph of the power function $y = \sqrt{x}$ right 100 units.

The domain is $x \geq 100$.

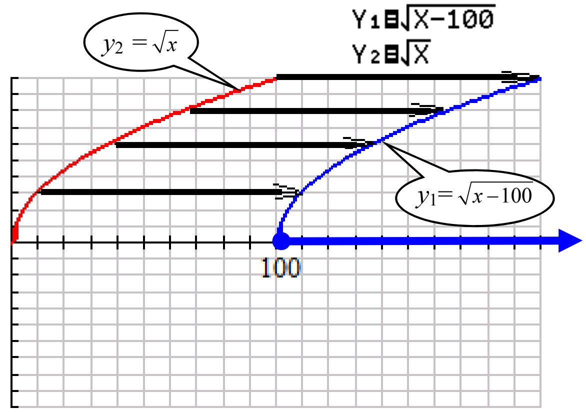
You can also write the domain $[100, \infty)$.

Use a table to confirm that 100 is included in the domain, as well as all reals larger than 100.

Values less than 100 cause the calculator to bail.

X	Y1
95	ERROR
96	ERROR
97	ERROR
98	ERROR
99	ERROR
100	0
101	1
102	1.4142
103	1.7321
104	2
105	2.2361

Y1=√(X-100)
Y2=√X

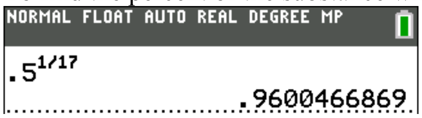


The viewing window is $0 \leq x \leq 200$ by $-10 \leq y \leq 10$, with Xscl = 10 and Yscl = 1.

The formula $f(x) = \sqrt{x-100}$ tells you that $f(x)$ is defined if the radicand $x - 100 \geq 0$.

When you solve this inequality, you have $x \geq 100$.

(c) If a substance decays according to the formula $P(t) = 200(0.5)^{t/17}$, where t is in minutes, its half-life is **17** minutes. Check by substitution.
To find the percent of the substance which decays each minute, first find the growth factor.



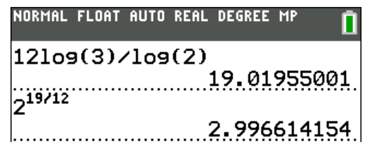
Thus $P(t) = 200(0.5)^{\frac{t}{17}} = 200(0.5^{\frac{1}{17}})^t = 200(0.96)^t$.
Since 96% of the substance is retained each minute, we have that 4% decays each minute.

(d) If a population with initial amount P_0 doubles every 12 years, it is modeled by $P(t) = P_0(2)^{\frac{t}{12}}$.

To find the tripling time, solve $P(t) = P_0(2)^{\frac{t}{12}} = 3P_0$.

Divide both sides by P_0 and take logarithms: $(2)^{\frac{t}{12}} = 3$

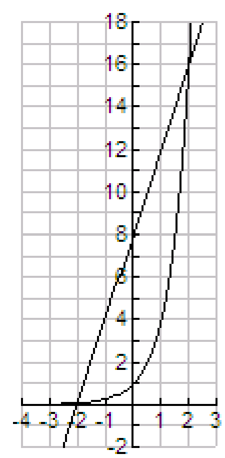
$$\log(2)^{\frac{t}{12}} = \log 3$$
$$\tfrac{t}{12}\log(2) = \log 3$$
$$t = \frac{12\log 3}{\log 2} \approx 19 \text{ years.}$$



We can check by substituting back into the original equation. $P(19) = P_0(2)^{\frac{19}{12}} \approx 3P_0$.

(e) $x = 2$ is a solution to the equation $4x + 8 = 4^x$ since

$$4x + 8 = 4^x$$
$$4(2) + 8 = 4^2$$
$$8 + 8 = 16$$



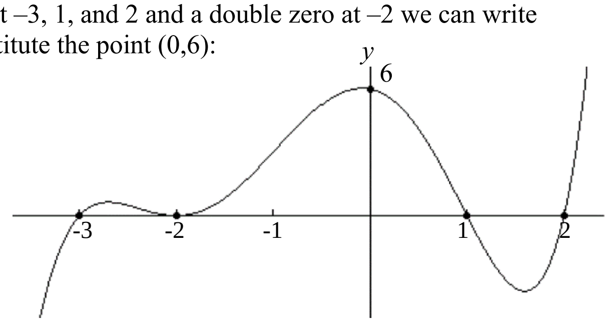
To solve $4x + 8 > 4^x$, we must find all solutions to $4x + 8 = 4^x$, which can only be solved graphically or numerically.
The solutions are $x = -1.98403$ and $x = 2$.

From the graph, the solution to $4x + 8 > 4^x$ are the values of x when the graph of $y = 4x + 8$ is ***above*** the graph of $y = 4^x$, which is $-1.98403 < x < 2$

38. Choice **C**. $h(x) = x^3$ The domain and range of $h(x) = x^3$ are all real numbers.

39. (a) The polynomial has formula $y = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$
Because the function has single zeros at –3, 1, and 2 and a double zero at –2 we can write $y = a(x-2)(x-1)(x+3)(x+2)^2$ Now substitute the point (0,6):

–



$$\left.\begin{array}{l} x=0 \\ y=6 \end{array}\right\} y=a(x-2)(x-1)(x+3)(x+2)^2$$

$$6=a(-2)(-1)(3)(2)^2$$

$$6=24a$$

$$a=\frac{6}{24}=\frac{1}{4}$$

Therefore, the polynomial is $f(x)=\frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$

To find $f(3)$, let $x=3$: $f(3)=\frac{1}{4}(3-2)(3-1)(3+3)(3+2)^2=\frac{1}{4}(1)(2)(6)(5)^2=75$

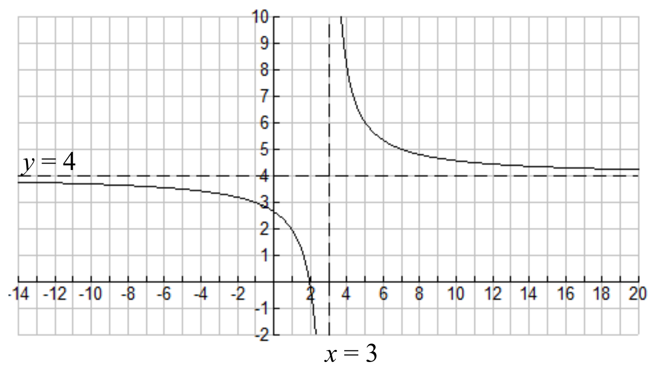
You could also use the table feature of a graphing calculator. Choice **B**.

Important:
You should check
with a graphing calculator
to be sure that the function is correct.

X	Y1
-4	-30
-3	0
-2	0
-1	3
0	6
1	0
2	0
3	75
4	378
5	1176
6	2880

This should match the given information provided.

(b) The rational function has the formula $y=\dfrac{4(x-2)}{(x-3)}$



Because the zeros of the function is 2,
we have $(x-2)$ as a factor of the numerator
since the function is 0 when the numerator is 0.

Since the vertical asymptote is $x=3$,
we have $(x-3)$ as a factor of the denominator.
(The vertical asymptotes are found where the denominator is 0 and the numerator is not).

So we can write $y=\dfrac{a(x-2)}{(x-3)}$.

Since the horizontal asymptote is $y=4$ and it is found by the ratio of the leading terms, we must have $a=4$.

Therefore the function must be $f(x)=\dfrac{4(x-2)}{(x-3)}$. Use a table feature to find Choice **C** is correct.

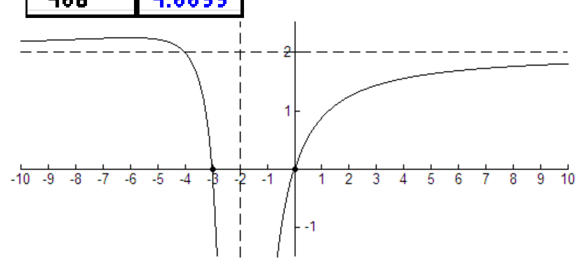
Alternatively, use the formula: $f(403)=\dfrac{4(403-2)}{(403-3)}=\dfrac{4\cdot 401}{400}=\dfrac{401}{100}=4.01$

```
Y1=4(X-2)/(X-3)
```

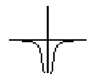
X	Y1
403	4.01
404	4.01
405	4.01
406	4.0099

$f(403)=4.01$

(c) The rational function has the formula $y=\dfrac{2x(x+3)}{(x+2)^2}$.



Because the zeros of the function are
0 and –3, the factors of the numerator are $x(x+3)$,
since the function is 0 when the numerator is 0.

There is one vertical asymptote at $x = -2$, so $(x + 2)$ is a factor of the denominator. However, the short run behavior near this asymptote looks like $y = k/x^2$ () so the factor must have a power of 2.

We can write $y = \dfrac{ax(x+3)}{(x+2)^2}$. Since the horizontal asymptote is $y = 2$, we must have $a = 2$.

Note: $y = \dfrac{ax(x+3)}{(x+2)^2} \approx \dfrac{ax^2}{x^2} = a$ as $x \to \pm\infty$ so $a = 2$.

Therefore, the rational function has the formula $y = \dfrac{2x(x+3)}{(x+2)^2}$.

Use a table to confirm:

X	Y1
-6	2.25
-5	2.2222
-4	2
-3	0
-2	ERROR
-1	-4
0	0
1	.88889
2	1.25
3	1.44
4	1.5556

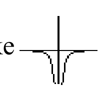
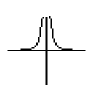
This should match the given information provided.

Use a table to find determine if $f(-1) = -4$, $f(1) = 1$, and $f(-6) = 2.25$
:

X	Y1	
-6	2.25	$f(-6) = 2.25$
-5	2.2222	
-4	2	
-3	0	
-2	ERROR	
-1	-4	$f(-1) = -4$
0	0	
1	.88889	$f(1) \neq 1$
2	1.25	
3	1.44	
4	1.5556	

Since only Choices A and C are true, Choice **D** is correct.

40. The equation is $y = \dfrac{8(x-4)}{(x-2)^2}$

Because there is a horizontal asymptote of $y = 0$, the degree of the numerator is less than the degree of the denominator. The numerator has a factor of $(x-4)^1$ since it has a single zero. Because the short run behavior near the vertical asymptote looks like  or 

the lowest degree possible for the denominator must be 2.

So it has a factor of $(x-2)^2$. It has the form $y = \dfrac{a(x-4)}{(x-2)^2}$,
and we can find a if we use the fact that when $x = 0$, $y = -8$:

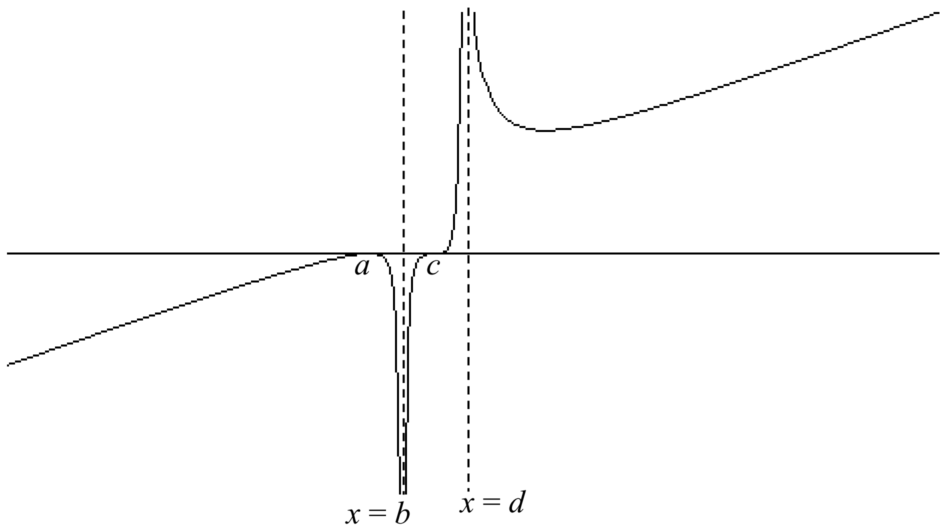
$$-8 = \frac{a(0-4)}{(0-2)^2}$$
$$-8 = \tfrac{-4}{4}a$$
$$a = 8$$

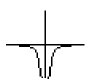
So $f(x)=\dfrac{8(x-4)}{(x-2)^2}$. To find $f(3)$, we let $x = 3$ and find y.

$$f(3)=\frac{8(3-4)}{(3-2)^2}=\frac{8(-1)}{1}=-8$$

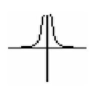
Alternatively, you can enter the formula in a grapher and use a table. Choice **B.**

41. The degree of the factor $(x-a)$ must be even since there is a bounce at the zero.



The degree of the factor $(x-b)$ must be even since the vertical asymptote appears as  near b.

The degree of the factor $(x-c)$ must be 3, 5, … since there is a chair at the zero.

The degree of the factor $(x-d)$ must be even since the vertical asymptote appears as  near d.

The long run behavior is the same as the power function $y = kx$, so the degree of the numerator must be one more than the degree of the denominator. Therefore, it must be Choice **B**.

42. I. Choice **C.** $y=B-Ax$ since it has a positive y-intercept (B) and slope is negative ($-A$).

II. Choice **C** $y=\log(x+A)$ since it is a shift of $y = \log x$ to the left A units. (Its vertical asymptote is at $x = -A$.)

III. Choice **A** $y=|x-A|$ since it is a shift of $y = |x|$ to the right A units. (Its minimum is when $x = A$.)

IV. Choice **C** $y=A(x+B)^2-C$ since the x-and y-coordinate coordinates of the vertex are negative and the parabola is concave up.

V. Choice **C** $y=-A(x+B)^5+C$ since it is a vertical reflection of $y=x^5$ combined with a horizontal shift to the left and a vertical shift up.

VI. Choice **D** $y=(1/A)^x$ since it is exponential decay.

VII. Choice **C** $y = \dfrac{A(x+B)}{x-C}$ since its vertical asymptote is $x = C$ with C positive, it has a horizontal asymptote $y = A$ with A positive, and a negative zero (at $-B$).

VIII. Choice **A** $y = \dfrac{A}{(x-B)^2} - C$ since it is a shift of $y = \dfrac{A}{x^2}$ to the right B units and down C units.

43. The average rate of change is $\dfrac{\Delta V}{\Delta t}$.

Find the change in time, Δt, and the change in volume, ΔV, over the intervals.

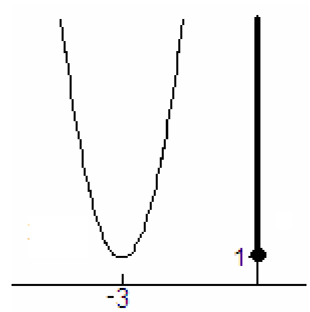
Δt	Time, t (min)	Volume, V (gal)	ΔV
	30	1075	
30 minutes	60	1150	75 gal
30 minutes	90	1225	75 gal
30 minutes	120	1300	75 gal

Then create ratios. The average rate of change will be the rate at which the water fills the pool.

$\dfrac{\Delta V}{\Delta t} = \dfrac{75\,\text{gal}}{30\,\text{min}} = 2.5$ gallons per minute.

The answer is Choice **E**.

44. The range is the set of all possible values of y.
The range of the function $y = 5x^2$ is all real numbers greater than equal to 0.
The function shown is a translation of $y = 5x^2$ up 1, so the range is $[1, \infty)$.
Notice on the graph to the right, values of y begin at $y = 1$ and increase forever.
Choice **B**.



45.

$$\log_b\left(\frac{x^3y^2}{\sqrt{w}}\right) = \log_b x^3 + \log_b y^2 - \log_b \sqrt{w}$$
$$= \log_b x^3 + \log_b y^2 - \log_b w^{1/2}$$
$$= 3\log_b x + 2\log_b y - \tfrac{1}{2}\log_b w$$

The correct answer is Choice **C**.

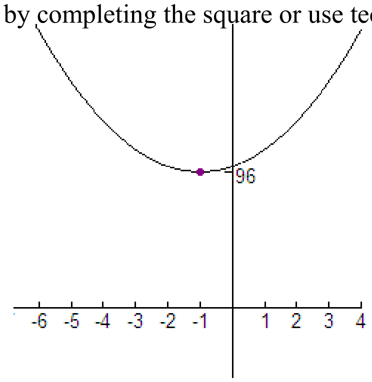
46.

$$25^x = 3^{600}$$
$$\ln 25^x = \ln 3^{600}$$
$$x \ln 25 = 600 \ln 3$$
$$x = \frac{600 \ln 3}{\ln 25} \approx 204.78$$

The correct answer is Choice **C**.

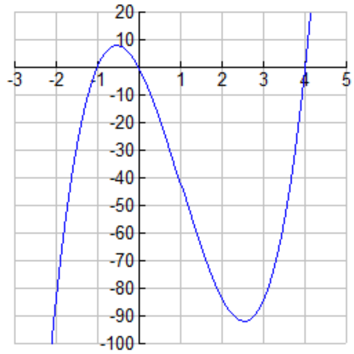
47. $(-1, 96)$ You can find the vertex by completing the square or use technology. Choice **D**.

Note: If you use technology to find the minimum of the graph, you would **not** report points such as $(-1.000001, 96)$ or $(-0.9999983, 96)$. The table feature, however, would show $(-1, 96)$.



$$
\begin{aligned}
y &= 4x^2 + 8x + 100 \\
&= 4(x^2 + 2x \quad) + 100 \\
&= 4(x^2 + 2x + 1 - 1) + 100 \\
&= 4[(x^2 + 2x + 1 - 1)] + 100 \\
&= 4[(x+1)^2 \quad - 1)] + 100 \\
&= 4 \cdot (x+1)^2 \quad - 4 \cdot 1 + 100 \\
&= 4(x+1)^2 + 96
\end{aligned}
$$

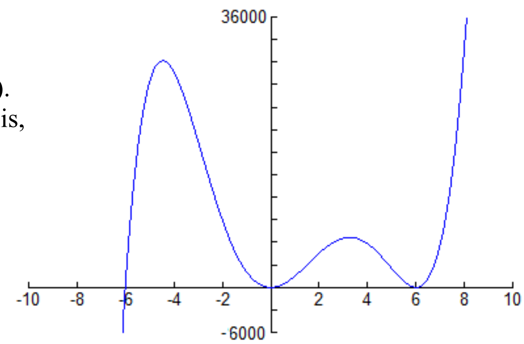
48. Sketch a graph. (Use the table feature to help you find a viewing window.) This suggests the function has zeros at −1, 0, and 4:



$$7(x^3 - 3x^2 - 4x) = 0$$
$$7x(x^2 - 3x - 4) = 0$$
$$7x(x-4)(x+1) = 0$$
$$7x = 0 \quad \| \quad x = 4 \quad \| \quad x = -1$$
$$x = 0$$

The correct answer is Choice **C**.

49. Sketch a graph of the polynomial $f(x) = 9x^2(x+6)(x-6)^2$ by hand (or use a grapher, but it's difficult to find a window). Determine the values of x for which f is above or on the x-axis, which is $x \geq -6$.



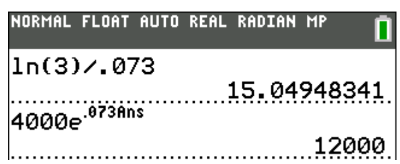
The correct answer is Choice **C**.

50. Solve $4{,}000e^{0.073t} = 12{,}000$.

$4000e^{0.073t} = 12{,}000$	Divide both sides by 4000 to get $e^{0.073t}$ all by itself.
$e^{0.073t} = 3$	Take natural logarithms of both sides.
$\ln e^{0.073t} = \ln 3$	Use the inverse property: $\ln e^{0.073t} = 0.073t$.
$0.073t = \ln 3$	Divide both sides by 0.073 to solve for t.

$$t = \frac{\ln 3}{0.073} \approx 15.05$$

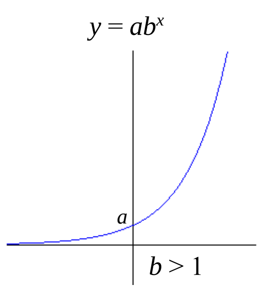
Choice **D**. TIP: Check by resubstituting:

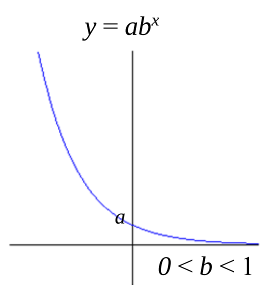


51. $\ln\left(\frac{1}{\sqrt{e^x}}\right) = \ln\left(\frac{1}{e^{x/2}}\right) = \ln\left(e^{-x/2}\right) = -\frac{x}{2}$ Choice **C**.

52. In general, the graph of $y = f(x) = ab^x$ increases for $b > 1$ and decreases for $0 < b < 1$ and has y-intercept $(0, a)$.

✓	I.	It increases if $b > 1$
	II.	It decreases if $b < 0$
✓	III.	It has y-intercept (0, 1) if $b > 0$.





The function $y = b^x$ is a special case, with $a = 1$. Therefore, Items I and III are correct. Choice **D**.

53. The graph of $y = 2 + \log(x - 1)$ is a horizontal shift 1 unit to the right and a vertical shift 2 units up of the graph of $y = \log(x)$.

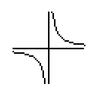
- Since the graph of $y = \log(x)$ has a vertical asymptote of $x = 0$, the graph of $y = 2 + \log(x - 1)$ has a vertical asymptote of $x = 1$.
- Since the domain of $y = \log(x)$ is the set of all real numbers $x > 0$, the domain of $y = 2 + \log(x - 1)$ is the set of all real numbers $x > 1$. Therefore it does **not** cross the x-axis at 1 and it never touches the y-axis.
- The graph of $y = 2 + \log(x - 1)$ passes through the point (2, 2):
  check: $x = 2, y = 2 \Rightarrow$ $y = 2 + \log(x - 1)$
  $2 = 2 + \log(2 - 1)$?
  $2 = 2 + \log(1)$?
  $2 = 2 + 0$? **YES**
- The range of the function $y = 2 + \log(x - 1)$ is all real numbers.

A graph of $y = 2 + \log(x-1)$ produced by technology in a standard window
$-10 \le x \le 10$
$-10 \le y \le 10$
can look misleading!

It is difficult for most technology to produce an accurate graph of a logarithm function.
Don't be deceived by a misleading graph.
Therefore Items I, III, and IV are correct.
Choice **E**.

✓ I. increases for all values of x in its domain.
~~II. crosses the x-axis at 1~~
✓ III. never touches the y-axis
✓ IV. passes through the point (2, 2).

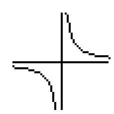
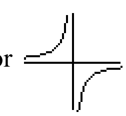
54. Since the vertical asymptote is $x = a$, the **denominator** must have $(x - a)$ as a factor.
Since the function has a single zero through the origin (0, 0), the **numerator** must be 0 when $x = 0$.

The short run behavior of the function near its vertical asymptote looks like  requiring the factor in the **denominator to be raised to an odd power**.

The equation $y = \dfrac{x}{x-a}$ is the only choice which meets these three criteria. Choice **C**.

.

55. As $x \to \infty$ or $x \to -\infty$, $f(x) = \dfrac{2ax}{(x-a)^2} \approx \dfrac{2ax}{x^2} = \dfrac{2a}{x}$.

In other words, the graph of $y = \dfrac{2ax}{(x-a)^2}$ and the graph of $y = \dfrac{2a}{x}$ have the same long run behavior. The graph of $y = \dfrac{2a}{x}$ has end behavior which looks like  or 
(depending on whether a is positive or negative).
In either case, as $x \to -\infty$ or as $x \to \infty$, the function approaches 0
The horizontal asymptote is $y = 0$. Choice **D**.

56. Since pH = –log C and pH = 2.1, we must solve the logarithmic equation $2.1 = -\log C$.

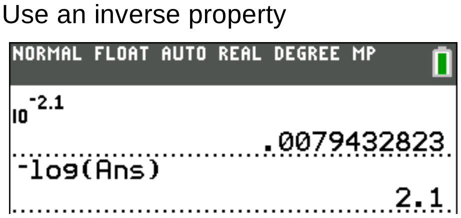
$-\log C = 2.1$ Multiply both sides by -1

$\log C = -2.1$ Make both sides a power of 10

$10^{\log C} = 10^{-2.1}$ Use an inverse property

$$C = 10^{-2.1} = \frac{1}{10^{2.1}} \approx 0.0008$$

Choice **B**.



57. To solve $\ln 2x^3 = 5$, exponentiate both sides to base e:

$e^{\ln 2x^3} = e^5$ Make both sides a power of e.

$2x^3 = e^5$ Use the inverse property.

The answer is Choice **D**.

58. To solve $\ln 2x^3 = 5$

$2x^3 = e^5$ From Question 57.

$x^3 = \frac{1}{2}e^5$ Divide both sides by 2.

$x^3 = \dfrac{e^5}{2}$

$x = \sqrt[3]{\dfrac{e^5}{2}}$ Take the cubed root of both sides

You can check by substitution: $\ln 2\left(\sqrt[3]{\dfrac{e^5}{2}}\right)^3 = \ln 2\left(\dfrac{e^5}{2}\right) = \ln e^5 = 5$. The answer is Choice **C**.

59. To solve $20 = 3e^x + 5$ first subtract 5 from both sides:

This gives us $15 = 3e^x$. The answer is Choice **D**.

60. To solve $20 = 3e^x + 5$

$3e^x - 15$ From Question 59.

$e^x = 5$ Divide both sides by 5.

$\ln e^x = \ln 5$ Take natural logs of both sides.

$x = \ln 5$ Use the inverse property.

You can check by substitution: $3e^{\ln 5} + 5 = 3 \cdot 5 + 5 = 20$. The answer is Choice **E**.

61. Use $P(1+\frac{r}{n})^{n\cdot t}$ with $P = 2200$, $r = 0.0382$, and $n = 4$. The balance in year t is $2200(1+\frac{0.0382}{4})^{4t}$.

Remember that 3.82 per cent is $\dfrac{3.82}{100} = 0.0382 = 3.82\%$.

The answer is Choice **C**.

TIP: To divide 3.82 by 100, move the decimal point of 3.82 two places to the left.
For example: **03.**82% becomes 0**.03**82

62. Since you are compounding continuously, use $Pe^{r\cdot t}$ with $P = 2200$, $r = 0.0382$. (See previous question.)
The balance in year t is $2200e^{0.0382t}$. Note: $2200e^{0.382t}$ grows at a continuous rate of 0.382 = 38.2 %.
Since the balance is none of the choices listed, the answer is Choice **E.**
TIP: To multiply 0.382 by 100, move the decimal point of 0.382 two places to the right.
For example: 0**.38**2 becomes **38.**2%.

63. The balance at year t of \$1000 compounded annually at 5% is $1000(1.05)^t$.

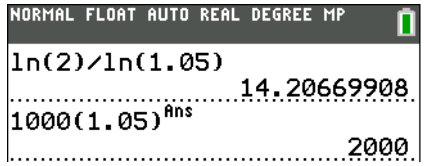
a) The amount in year 7, reported to the nearest \$0.01, is $1000(1.05)^7 \approx$ **\$1407.10**.

b) To find the total percent by which the account increased at the end of the 7 year period, first find the growth factor b, where $1000b = 1407.10$. Divide both sides by 1000.

c) The doubling time of the account is the value of t for which $1000(1.05)^t = 2000$.

Solve $(1.05)^t = 2$

$$\ln 1.05^t = \ln 2$$
$$t \ln 1.05 = \ln 2$$
$$t = \frac{\ln 2}{\ln 1.05} \approx 14.2 \text{ years}$$



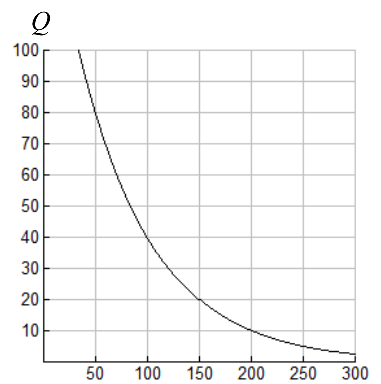
64. a) Since each value of Q is halved every 50 years, work backwards to find Q_0 by doubling 80 so $Q_0 = \mathbf{160}$.

b) Complete the next three rows of the table by taking half of the previous row's output. Check it matches the graph.

c) The half-life is the time it takes to decay by half, or **50 years**.

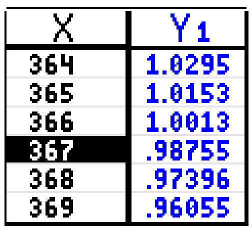
d) $Q = f(t) = 160(0.5)^{t/50}$ (Similar to Question 37c in this packet. A longer method to get this formula is use the procedure in Questions 25-26 of this packet.)

t, years	$Q = f(t)$, mg
0	**160**
50	80
100	40
150	20
200	**10**
250	**5**
300	**2.5**



e) Solve $160(0.5)^{t/50} = 1$ with logarithms or a table or graphically.

Using a table:



X	Y1
364	1.0295
365	1.0153
366	1.0013
367	.98755
368	.97396
369	.96055

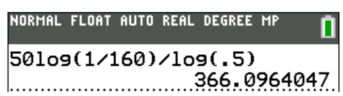
For $t = 366$ and earlier Q is above 1 mg.

For $t \geq 366$ and higher Q is **below** 1 mg.

In **367 years** it will first be below 1 mg.

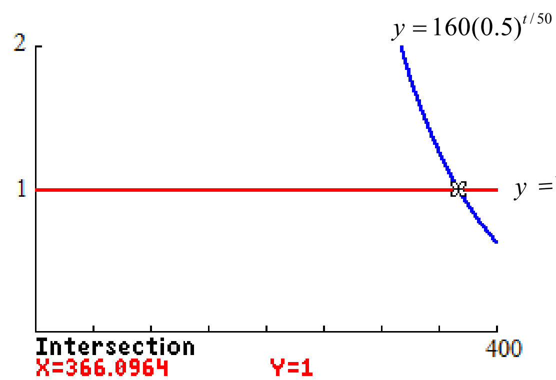
Using logarithms (analytically):

$$160(0.5)^{t/50} = 1$$
$$(0.5)^{t/50} = \tfrac{1}{160}$$
$$\log(0.5)^{t/50} = \log \tfrac{1}{160}$$
$$\tfrac{t}{50} \log 0.5 = \log \tfrac{1}{160}$$
$$t = \frac{50\log(1/160)}{\log 0.5} \approx 366.0964$$



At 366 years the amount is above 1 mg.
So in **367 years it** will first be below 1 mg.

Graphically: See the graph to the right.



f) Every year the amount decays by 1.4%.
This is similar to Question 37c.

$Q = 160(0.5)^{t/50} = 160(0.5^{\frac{1}{50}})^t \approx 160(0.986)^t$

Since 96.6% is kept each year, 1.4% is lost each year.

65. The function $f(x)=\dfrac{4}{x^2}$ takes any input and returns 4 divided by the square of the input.

We can replace x by a placeholder, such as an empty box, i.e.. $f(\boxed{})=\dfrac{4}{(\boxed{})^2}$

If f takes the function $g(x)=\sqrt{x^2+4}$ as an input, then we have

$$f(\boxed{g(x)})=\frac{4}{\left(\boxed{\sqrt{x^2+4}}\right)^2}$$

$$=\frac{4}{x^2+4}$$

This is as simplified as possible. The answer is Choice **A**.

66. For the function $f(x)=\dfrac{\sqrt{x+1}}{2}$ we can replace x by a placeholder, such as an empty box, i.e..

$$f(\boxed{})=\frac{\sqrt{\boxed{}+1}}{2}$$

If f takes the function $g(x)=x^2+3$ as an input, then we have

$$f(\boxed{g(x)})=\frac{\sqrt{\boxed{x^2+3}+1}}{2}$$

$$=\frac{\sqrt{x^2+3+1}}{2}$$

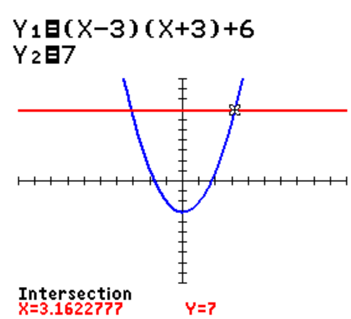
$$=\frac{\sqrt{x^2+4}}{2}$$

This is as simplified as possible. The answer is Choice **B**.

67. The answer is Choice **B**. William's answer is incorrect. His error was in Step 2.
It is ***false*** to conclude that $A\cdot B=1 \Leftrightarrow A=1$ or $B=1$.
There are infinitely many ways for a product of two numbers $A\cdot B=1$.
(For example, $A=\frac{1}{2}, B=2$ is one possibility. $A=\frac{1}{\sqrt{\pi}}, B=\sqrt{\pi}$ is another.)
Note: William would have been correct if he had concluded $A\cdot B=0 \Leftrightarrow A=0$ or $B=0$,
which is called the "Zero Factor Property".
Had William checked his answer, he would have seen if

$$x=4 \quad \Rightarrow (4-3)(4+3)+6=(1)(7)+6=13\neq 7$$

$$x=-2 \Rightarrow (-2-3)(-2+3)+6=(-5)(1)+6=1\neq 7$$

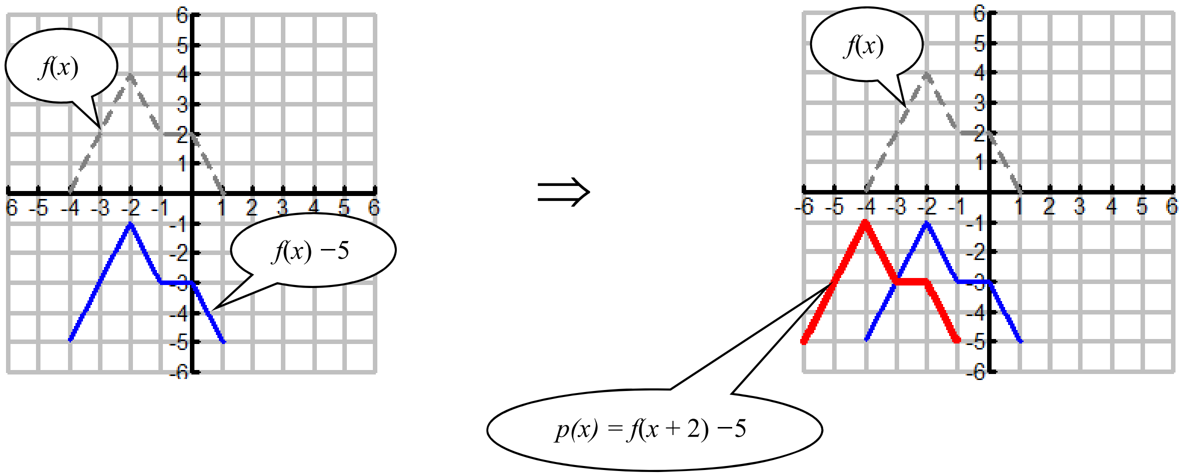


Had William used a grapher to check, he would have seen there are two solutions but they are not integers. The correct solution can be solved analytically by expanding:

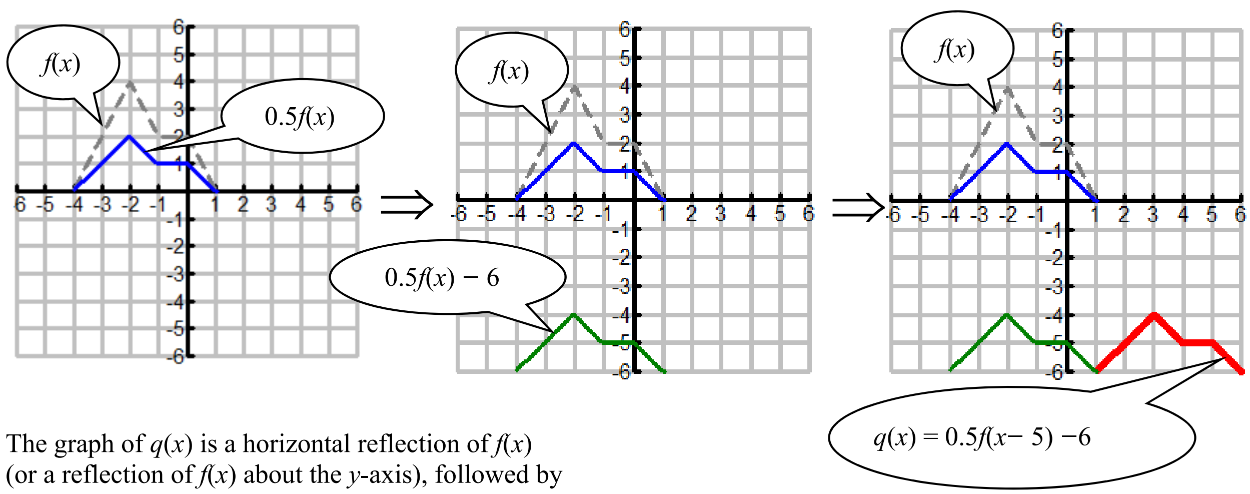
$$(x-3)(x+3)+6=7$$
$$(x-3)(x+3)=1$$
$$x^2-9=1$$
$$x^2=10$$
$$x=\pm\sqrt{10}$$

The **exact** solutions are $x=-\sqrt{10}$ and $x=\sqrt{10}$. (Note: $\pm$ 3.1622777 is the solution approximated to a mere 7 places and is not exact. To 14 places the value of $\sqrt{10}\approx 3.16227766016838$, so any decimal representation is approximate.)

68. The graph of $p(x)$ is a vertical shift of $f(x)$ down 5 and horizontal shift left 2.
The transformation is $p(x) = f(x + 2) - 5$



69. The graph of $q(x)$ is a vertical compression of $f(x)$ by a factor of $\frac{1}{2}$,
followed by a vertical shift down 6 and horizontal shift 5 right.
The transformation is $q(x) = 0.5f(x - 5) - 6$



70. The graph of $q(x)$ is a horizontal reflection of $f(x)$
(or a reflection of $f(x)$ about the y-axis), followed by
a vertical shift down 4. The transformation is $r(x) = f(-x) - 4$.