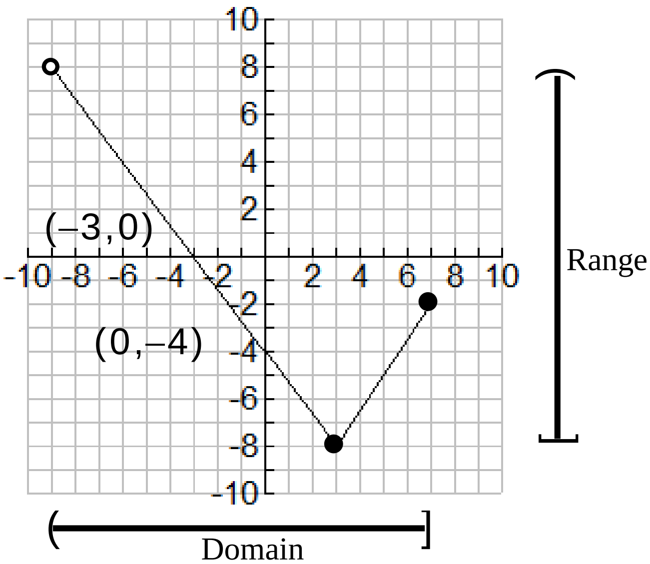
Full Key for MA 11100 Review for Final Exam

<u>For problems #1-6.</u> Shown is the graph of function $y = f(x)$.



1. A $(-9, 7]$ The Domain is the set of all *x*'s.
2. D $[-8, 8)$ The Range is the set of all *y*'s.
3. C $(-3, 0)$ The *x*-intercept is the point where the graph crosses the *x*-axis.
4. B $(0, -4)$ The *y*-intercept is the point where the graph crosses the *y*-axis.
5. D -4 When $x = 0$, $y = f(x) = -4$
6. B -3 When $y = f(x) = 0$, $x = -3$
7. Which of the graphs below are functions? Do the Vertical Line Test. If any vertical line intersects the graph in more than one point, the graph is not that of a function.

I) Yes	II) No	III) No	IV) No
V) Yes	VI) No	VII) Yes	VIII) Yes

8. B -3 is included, so use [on -3; 9 is not included, so use) on 9.
9. C $x > -\frac{14}{13}$

$$9 - 5(x + 7) + 2x < 4 - 2(8 - 5x)$$
$$9 - 5x - 35 + 2x < 4 - 16 + 10x$$
$$-3x - 26 < -12 + 10x$$
$$-3x - 10x < -12 + 26$$
$$-13x < 14$$
$$x > -\frac{14}{13}$$

10. C From the graph we see that the line passes through $(0,3)$ and $(2,0)$. Thus, $b=3$ and $m=\frac{0-3}{2-0}=-\frac{3}{2}$. The equation, $y=mx+b$, becomes $y=-\frac{3}{2}x+3$.

11. E Letting $x=0$ represent the year 2005, we have the point $(0,17)$. Since the revenue increases by \$1.2 million each year, $m=1.2$. $R=1.2x+17$. For 2014, let $x=9$, $R=1.2(9)+17=\$27.8$ million.

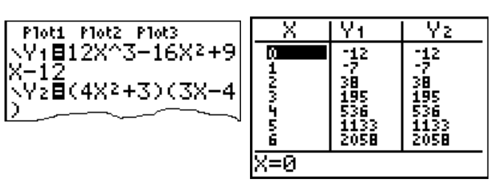
12. E $m=\frac{y_2-y_1}{x_2-x_1}=\frac{13-(-7)}{-4-2}=\frac{20}{-6}=-\frac{10}{3}$

13. B
$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-5&=3(x-(-2))\\ y-5&=3x+6\\ y&=3x+11 \end{aligned}$$

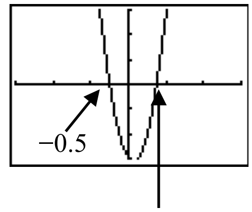
14. E Solve $4x-3y=11$ for y first. $y=\frac{4}{3}x-\frac{11}{3}$. The slope of the given line is $m=\frac{4}{3}$. The slope of the perpendicular line is $m=-\frac{3}{4}$. Using the point-slope form of a line, we have
$$\begin{aligned} y-7&=-\frac{3}{4}(x-(-8))\\ y-7&=-\frac{3}{4}x-6\\ y&=-\frac{3}{4}x+1 \end{aligned}$$

15. D Since the line is increasing (rising) the slope *m* must be positive, so $m>0$. The *y*-intercept is below the origin, so *b* must be negative, thus $b<0$.

16. D Check first if you can remove a GCF. None to be had here. Since you have 4 terms, factor by grouping.
$$\begin{aligned} 12x^3-16x^2+9x-12&=4x^2(3x-4)+3(3x-4)\\ &=(4x^2+3)(3x-4) \end{aligned}$$
Finally check if each of the factors can be further factored. Not possible, so we're done. Check by multiplying out. You can also check with the table to see if $y=12x^3-16x^2+9x-12$ and $y=(4x^2+3)(3x-4)$ are the same.



17. B To help factor the expression $8x^2 - 2x - 3$, enter it in a grapher. The graph seems to have a zero at $-\frac{1}{2}$. This suggests it would have a factor of $(2x+1)$, which can be helpful in deducing the remaining factor since $4 \cdot 2 = 8$ and $1 \cdot -3 = -3$. So the other zero would be $\frac{3}{4}$.



$$8x^2 - 2x - 3 = (2x+1)(\boxed{?}x + \boxed{?})$$
$$= (2x+1)(\boxed{4}x + \boxed{-3}) = (2x+1)(4x-3)$$

18. A Check first if you can remove a GCF. We have −4 common to all terms.

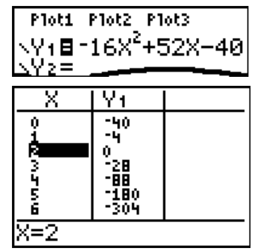
$-16x^2 + 52x - 40 = -4(4x^2 - 13x + 10)$ Take out a GCF first.

Notice it has 2 as a zero, so it should have $(x-2)$ as a factor.

Check: $= -4(4x-5)(x-2)$

$$\begin{array}{r} -5x \\ \underline{-8x} \\ -13x \end{array}$$

Since 10 is positive, we need two numbers of the same sign. Since they add up to a middle term which is negative, both numbers are negative.



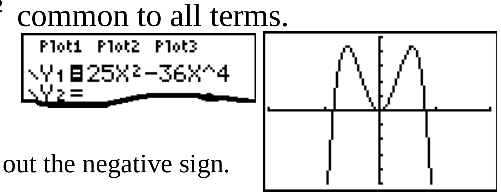
Finally check if each of the factors can be further factored. Not possible, so we're done. You can check with a grapher that $y = -16x^2 + 52x - 40$ and $y = -4(4x-5)(x-2)$ have the same table.

19. C Check first if you can remove a GCF. We have x^2 common to all terms.

$25x^2 - 36x^4 = x^2(25 - 36x^2)$

$= x^2(5-6x)(5+6x)$

OR $= -x^2(6x-5)(6x+5)$ We could also factor out the negative sign.



If you graph $y = 25x^2 - 36x^4$, you can see it has zeros at 0, and two values symmetric about the origin which are close to ± 1. These are 5/6 and –5/6.

20. B $7x(3x+5) = 0$ Set 7*x* = 0 and 3*x* + 5 = 0. Solving for *x*, we have *x* = 0 and $x = -\frac{5}{3}$.

21. E The "parent" absolute value function is shifted left 6, up 3, so its vertex is (–6, 3).

22. D $2x + 3y = 7 \Rightarrow (4)(2x+3y) = (4)(7)$

$-x + 4y = 24 \Rightarrow (-3)(-x+4y) = (-3)(24)$

$$\begin{array}{l} 8x + 12y = 28 \\ \underline{3x - 12y = -72} \\ 11x \quad\quad = -44 \\ \quad\quad x = -4 \end{array}$$

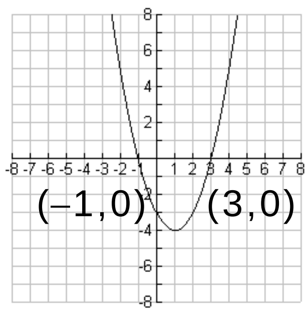
23-24. $x = 11$ $y = -15$

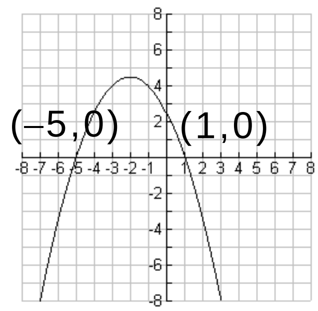
$\begin{cases} 4x - 3y = 89 \\ 5x + 4y = -5 \end{cases}$ multiply by 4, multiply by 3 $\begin{cases} 16x - 12y = 356 \\ 15x + 12y = -15 \end{cases}$ $31x = 341$ $x = 11$

$5x + 4y = 5(11) + 4y = -5$ $4y = -5 - 55 = -60$ $y = -15$

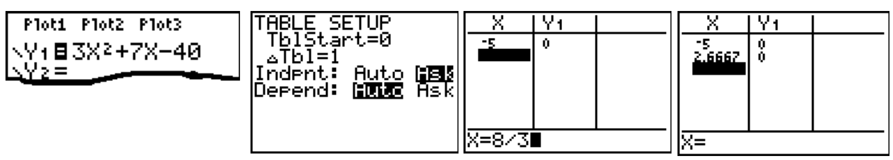
25. F –1 or 3 When $y = f(x) = 0$, $x = -1$ or $x = 3$

26. D –5 or 1 When $y = f(x) = 0$, $x = -5$ or $x = 1$





27. C One possibility is $f(x) = (x+5)(3x-8) = 3x^2 + 7x - 40$. Check with a table. Use the ASK Feature.



28. B One possibility is $f(x) = (x+4)(x-5) = x^2 - x - 20$. Check with a table. Use the ASK Feature, as in #27.

29. D 1.84 sec $h(t) = -4.9t^2 + 9t + 50 = 50$ $-4.9t^2 + 9t = 0$

$-t(4.9t - 9) = 0$ $-t = 0$ or $4.9t - 9 = 0$ $t = 0$ or $t = \frac{9}{4.9} \approx 1.84$

0 seconds is the time when it was thrown, so it will return to that same height at 1.84 seconds.

30. D If the width is w, then the length is $l = 4w + 3$. The area is $lw = A$, or $(4w+3)w = 32.5$

$4w^2 + 3w - 32.5 = 0$

By the quadratic formula, we have

$$w = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-32.5)}}{2(4)} = \frac{-3 \pm \sqrt{529}}{8}$$

$$= \frac{-3 \pm 23}{8}$$

Choosing the positive value for w yields

$$w = \frac{-3+23}{8} = 2.5 \text{ meters}$$

$$l = 4(2.5) + 3 = 13 \text{ meters}$$

31. C $9x^2 = 64x$

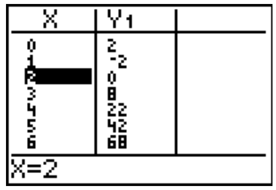
$9x^2 - 64x = 0$

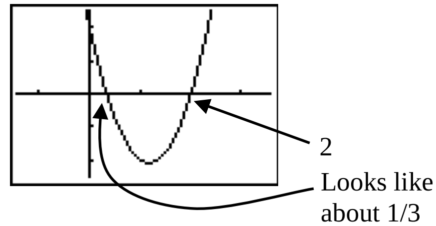
$x(9x - 64) = 0$

$x = 0$ or $x = 64/9$

32. E $x = \pm\frac{\sqrt{15}}{3}$ $9x^2 = 15$ $x^2 = \frac{15}{9}$ $x = \pm\frac{\sqrt{15}}{\sqrt{9}} = \pm\frac{\sqrt{15}}{3}$

33. B From the rational numbers given as choices, you get the clue this factors.
Enter the function in Y= to see 2 is a zero.





$3x^2 - 7x + 2 = 0$

$(3x - 1)(x - 2) = 0$

$3x - 1 = 0$ or $x - 2 = 0$

$x = \frac{1}{3}$ or $x = 2$

34. C

$x^2 + 12x \quad = 7$

$\left[\frac{1}{2}(12)\right]^2 = [6]^2 = 36$

$x^2 + 12x + 36 = 7 + 36$

$(x + 6)^2 = 43$

$x + 6 = \pm\sqrt{43}$

$x = -6 \pm \sqrt{43}$

35. A Write in standard form: $3x^2 + 5x - 7 = 0$. Use $a = 3, b = 5, c = -7$.

$b^2 - 4ac = 25 - 4(3)(-7) = 25 + 84 = 109$ so $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{109}}{6}$.

36. B Take square roots of both sides. $(2x - 7)^2 = 6$

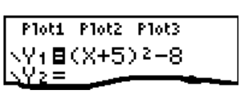
$2x - 7 = \pm\sqrt{6}$

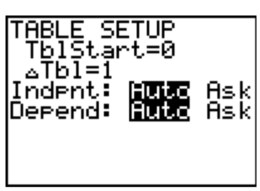
$2x = 7 \pm \sqrt{6}$

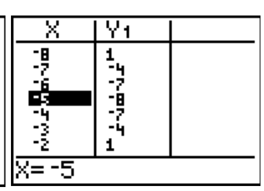
$x = \frac{7 \pm \sqrt{6}}{2}$

37. E $f(x) = (x + 5)^2 - 8$ is a translation of $y = x^2$ shifted left 5 and down 8. So its vertex is $(-5, -8)$.

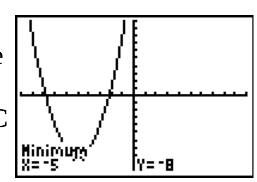
Other ways:







Or use **2nd CALC**



To find the y-intercept, let $x = 0$, then $y = (0 + 5)^2 - 8 = 25 - 8 = 17$. The y-intercept is $(0, 17)$.

38. B $f(x) = 3x^2 - 11x - 20 = (3x+4)(x-5)$

Setting each factor equal to zero yields $x = -\frac{4}{3}$ and $x = 5$. The x-intercepts are $\left(-\frac{4}{3}, 0\right)$ and $(5, 0)$.

39. C The vertex, $(h,k) = (1,2)$. Using $y = a(x-h)^2 + k$, we have $y = a(x-1)^2 + 2$. To find a, use the y-intercept $(0,3)$.

$y = a(x-1)^2 + 2$

$3 = a(0-1)^2 + 2$

$3 = a(1) + 2$

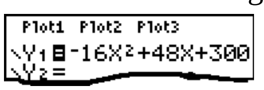
$1 = a$

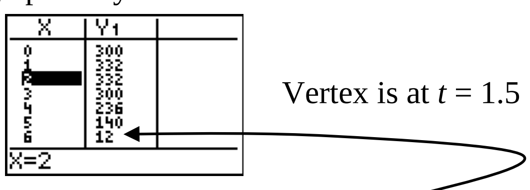
Thus, $y = 1(x-1)^2 + 2 = (x-1)^2 + 2$

40. D h and k are negative, a is positive.
The parabola opens up, so $a > 0$. The vertex (h, k) is in Quadrant III (–, –).

41. C a and k are negative, h is positive.
The parabola opens down, so $a < 0$. The vertex (h, k) is in Quadrant IV (+, –).

42. D You can solve this graphically. Use the table to find a window.





Vertex is at $t = 1.5$

At $t = 6$ the rock is still above ground (12 ft, in fact). So Choice D is the only plausible option.

You can also find it algebraically using the quadratic formula, but this would take much longer and could lead to errors. Divide both sides by -4: $4t^2 - 12x - 75 = 0$.
Use $a = 4,\ b = -12,\ c = -75$.

$b^2 - 4ac = 144 - 4(4)(-75) = 1344$ so $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{1344}}{8}$.

Use the positive solution $t = \frac{12 + \sqrt{1344}}{8} \approx 6.08258 \approx 6.1$ seconds.

43. B $\frac{x^{-5}x^{28}}{x^{-10}} = \frac{x^{10}x^{28}}{x^5} = x^5 x^{28} = x^{28+5} = x^{33}$

44. B $-x^{\frac{7}{9}} = -\sqrt[9]{x^7}$

45. C $x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{x^2}}$

46. Simplify: $\frac{10a^{10}b^{-10}c^5}{-15a^{-5}b^{20}c^{-5}} = \frac{-2}{3}a^{10-(-5)}b^{-10-20}c^{5-(-5)} = \frac{-2}{3}a^{15}b^{-30}c^{10} = -\frac{2a^{15}c^{10}}{3b^{30}}$

or: $\frac{10a^{10}b^{-10}c^5}{-15a^{-5}b^{20}c^{-5}} = \frac{-2a^{10}a^5c^5c^5}{3b^{20}b^{10}} = -\frac{2a^{15}c^{10}}{3b^{30}}$

47. Simplify: $\left(\frac{64x^{18}y^4}{125x^{-3}y}\right)^{\frac{1}{3}} = \left(\frac{64x^{18}x^3y^4}{125y}\right)^{\frac{1}{3}} = \left(\frac{64x^{21}y^3}{125}\right)^{\frac{1}{3}} = \frac{64^{\frac{1}{3}}x^{\frac{21}{3}}y^{\frac{3}{3}}}{125^{\frac{1}{3}}} = \frac{4x^7y}{5}$

48. C $\left(16x^{10}\right)^{\frac{3}{2}} = \left(\sqrt{16x^{10}}\right)^3$

$= \left(4x^5\right)^3 = 64x^{15}$

49. D $\frac{x^{\frac{1}{4}}x^{\frac{5}{4}}}{x^{\frac{7}{4}}} = \frac{x^{\frac{6}{4}}}{x^{\frac{7}{4}}} = \frac{x^{\frac{6}{4}}}{x^{\frac{6}{4}} \cdot x^{\frac{1}{4}}} = \frac{1}{x^{\frac{1}{4}}}$

50. A $\sqrt[5]{-32x^{15}y^{30}} = \sqrt[5]{(-2)^5x^{15}y^{30}} = -2x^{\frac{15}{5}}y^{\frac{30}{5}} = -2x^3y^6$

51. C $\sqrt[3]{64x^9y^{21}} = \sqrt[3]{4^3x^9y^{21}} = 4x^{\frac{9}{3}}y^{\frac{21}{3}} = 4x^3y^7$

52. C $\frac{x^{18}}{6y^8}$ $\sqrt{\frac{x^{36}}{36y^{16}}} = \frac{x^{\frac{36}{2}}}{\sqrt{36}y^{\frac{16}{2}}} = \frac{x^{18}}{6y^8}$

53.C $\sqrt[4]{x}$ $\sqrt[12]{x^3} = x^{\frac{3}{12}} = x^{\frac{1}{4}} = \sqrt[4]{x}$

54. E $\sqrt[12]{x^{11}}$ $\sqrt[3]{x^2} \cdot \sqrt[4]{x} = x^{\frac{2}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{2}{3}+\frac{1}{4}} = x^{\frac{8}{12}+\frac{3}{12}} = x^{\frac{11}{12}} = \sqrt[12]{x^{11}}$

55. C $4x^4 + 12x^3 - x^2 - 4x + 20$

$5x^4 + 10x^3 - 7x + 8 - x^4 + 2x^3 - x^2 + 3x + 12 =$
$5x^4 - x^4 + 10x^3 + 2x^3 - x^2 - 7x + 3x + 8 + 12 =$
$4x^4 + 12x^3 - x^2 - 4x + 20$

56. A Set the denominator $x^2 - 25 = (x+5)(x-5) = 0$ to find that 5 and −5 make the function undefined. The numerator does not affect the excluded values or the domain.

57. B Set the denominator $x^2 - 36 = (x+6)(x-6) = 0$ to find that 6 and −6 make the function undefined. The domain is all real numbers except these two values.

58. A

$$\frac{18-2x}{x^2-81}=\frac{-2x+18}{x^2-81}$$
$$=\frac{-2(x-9)}{(x-9)(x+9)}=-\frac{2}{x+9}$$

59. D

$$\frac{x^2-9}{5x-35}\cdot\frac{x-7}{x-3}=\frac{(x+3)(x-3)}{5(x-7)}\cdot\frac{x-7}{x-3}$$
$$=\frac{(x+3)\cancel{(x-3)}}{5\cancel{(x-7)}}\cdot\frac{\cancel{x-7}}{\cancel{x-3}}$$
$$=\frac{x+3}{5}$$

60. D

$$\frac{x-4}{2x}\div\frac{x^2-16}{3x^2}=\frac{x-4}{2x}\cdot\frac{3x^2}{x^2-16}$$
$$=\frac{x-4}{2x}\cdot\frac{3x^2}{(x-4)(x+4)}$$
$$=\frac{3x\cdot x\cdot(x-4)}{2x(x-4)(x+4)}$$
$$=\frac{3\cancel{x}\cdot x\cdot\cancel{(x-4)}}{2\cancel{x}\cancel{(x-4)}(x+4)}$$
$$=\frac{3x}{2(x+4)}$$

61. C

$$\frac{7x-9}{x^2+x-6}-\frac{1}{x-2}=\frac{7x-9}{(x-2)(x+3)}-\frac{1}{x-2}$$
$$=\frac{7x-9}{(x-2)(x+3)}-\frac{1}{(x-2)}\cdot\frac{(x+3)}{(x+3)}$$
$$=\frac{7x-9\quad+\quad-1\cdot(x+3)}{(x-2)(x+3)}$$
$$=\frac{7x-9\quad-x-3}{(x-2)(x+3)}$$
$$=\frac{6x-12}{(x-2)(x+3)}$$
$$=\frac{6(x-2)}{(x-2)(x+3)}$$
$$=\frac{6\cancel{(x-2)}}{\cancel{(x-2)}(x+3)}$$
$$=\frac{6}{(x+3)}$$

62. B $\dfrac{x}{x+4}-6=0$

$$\frac{x}{x+4}=6$$

$$(x+4)\left(\frac{x}{x+4}\right)=6(x+4)$$

$$x=6x+24$$

$$-5x=24$$

$$x=-\frac{24}{5}$$

63. D $\dfrac{x}{x-5}=\dfrac{7}{x-5}+\dfrac{1}{3}$ Check first for restrictions on *x*: x can't be 5

Multiply all terms by $3(x-5)$

$$\cancel{3(x-5)}\cdot\frac{x}{\cancel{x-5}} = \cancel{3(x-5)}\cdot\frac{7}{\cancel{x-5}} + \cancel{3}(x-5)\cdot\frac{1}{\cancel{3}}$$

$$3x=21+x-5$$

$$3x=x+16$$

$$2x=16$$

$$x=8$$

We check in the original and it is not a restricted value so *x* = 8.

64. B $\dfrac{1}{x+5}+\dfrac{7}{22x+20}=\dfrac{x+13}{22x+20}$ Check first for restrictions on *x*. $\dfrac{1}{x+5}+\dfrac{7}{2(11x+10)}=\dfrac{x+13}{2(11x+10)}$

Restrictions: x can't be -5 and $-\dfrac{10}{11}$

$$(22x+20)\cancel{(x+5)}\cdot\frac{1}{\cancel{x+5}}+(\cancel{22x+20})(x+5)\cdot\frac{7}{(\cancel{22x+20})}=(\cancel{22x+20})(x+5)\cdot\frac{x+13}{\cancel{22x+20}}$$

$$22x+20+7(x+5)=(x+5)(x+13)$$

$$22x+20+7x+35=x^2+18x+65$$

$$29x+55=x^2+18x+65$$

$$0=x^2-11x+10$$

$$0=(x-10)(x-1)$$

We check in the original and neither of these is a restricted value. Therefore *x* = 1 and 10.

65. E $4a^2b^4c^8\sqrt{5b}$ $\sqrt{80a^4b^9c^{16}}=\sqrt{16a^4b^8c^{16}}\cdot\sqrt{5b}=4a^2b^4c^8\sqrt{5b}$

66. D $\sqrt{3x-7}=36$

$3x-7=36^2$

$3x-7=1296$

$3x=1303$

$x=\dfrac{1303}{3}\approx 434.3$ This is in the interval $300<x<500$

67. C $\sqrt[3]{x+10}+9=-1$

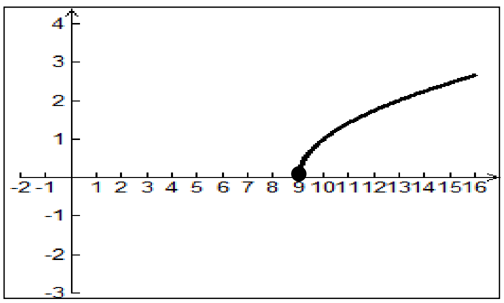
$\sqrt[3]{x+10}=-10$

$\left(\sqrt[3]{x+10}\right)^3=(-10)^3$

$x+10=-1000$

$x=-1010$

68. B Graphing $y_1=\sqrt{x-9}$ we see that it has an *x*-intercept at $(9,0)$ and no *y*-intercept



69. D $[9,\infty)$ $y=f(x)=\sqrt{x-9}$. See the graph for #68. The Domain is all real numbers $x\geq 9$.

70. D $4\sqrt{5}$ $\sqrt{(1--3)^2+(-6-2)^2}=\sqrt{(4)^2+(-8)^2}=\sqrt{16+64}=\sqrt{80}=\sqrt{16}\sqrt{5}=4\sqrt{5}$