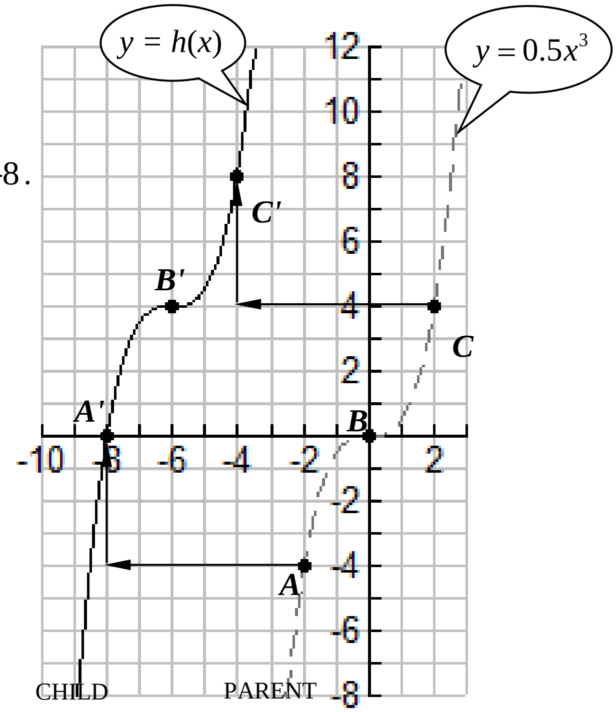
1. **a.** Horizontal shift 6 left and vertical shift 4 up.
Notice B' is $(-6, 4)$ and B is $(0, 0)$.

b. $h(x) = 0.5(x + 6)^3 + 4$ (Enter in a grapher to check.)

c. Use the graph. Notice A' to see $h(x)$ crosses the x-axis at -8.
Check with the formula.
If $x = -8$, $h(x) = 0.5(x + 6)^3 + 4$
$= 0.5(-8 + 6)^3 + 4$
$= 0.5(-2)^3 + 4$
$= 0.5(8) + 4 = 0.$

d. Use the formula. It crosses the y-axis when $x = 0$.
$h(0) = 0.5(0 + 6)^3 + 4 = 112$. You can also use the table.

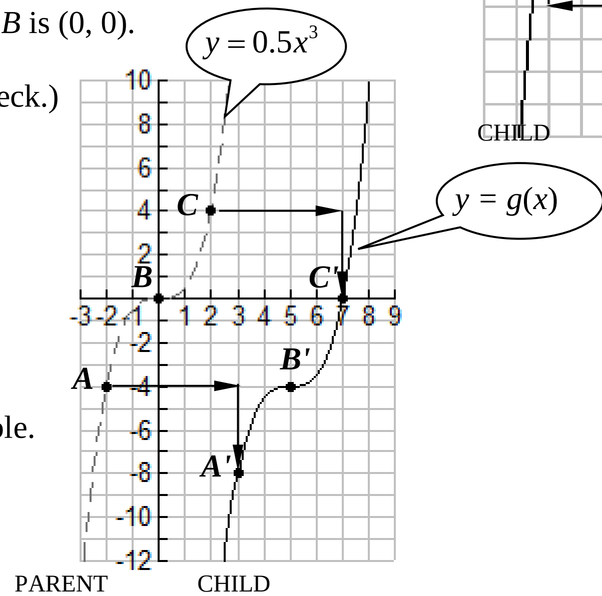


2. **a.** Horizontal shift 5 right and vertical shift 4 down.
Notice B' is $(5, -4)$ and B is $(0, 0)$.

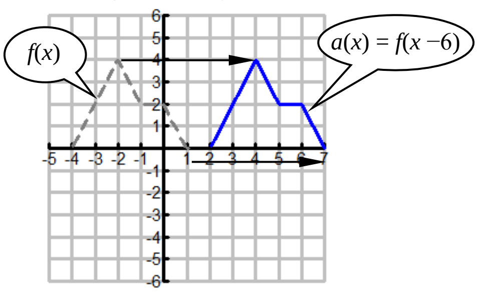
b. $g(x) = 0.5(x - 5)^3 - 4$
(Enter in a grapher to check.)

c. Notice C' to see $g(x)$
crosses the x-axis at 7.

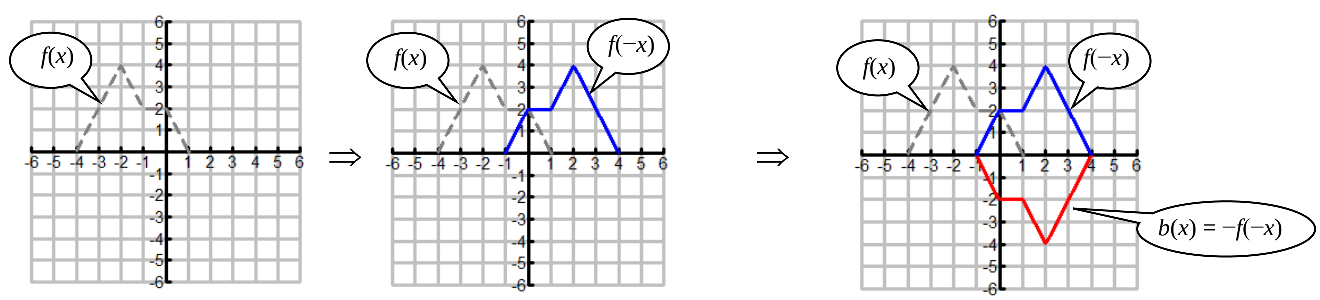
d. Use the formula.
It crosses the
y-axis when $x = 0$.
$x = 0$.
$g(0) = 0.5(0 - 5)^3 - 4$
$= -66.5$
You can also use the table.



3. **a.** The graph of $y = a(x)$ is a horizontal shift of the graph of $y = f(x)$ to the right 6 so $a(x) = f(x - 6)$.

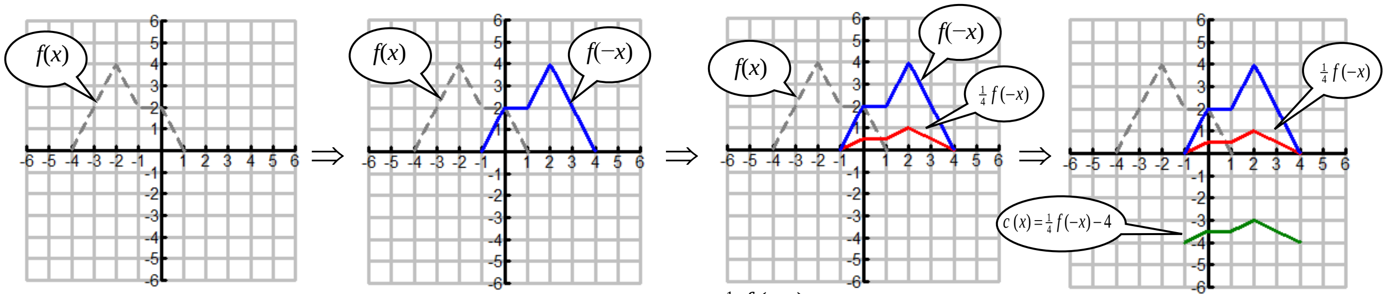


b. The graph of $y = b(x)$ is a horizontal and vertical reflection of the graph of $y = f(x)$ so $b(x) = -f(-x)$.



$f(-x)$ is a horizontal reflection of $f(x)$. $-f(-x)$ is a vertical reflection of $f(-x)$.

c. The graph of $y = c(x)$ is a horizontal reflection, followed by a vertical compression by a factor of $\frac{1}{4}$, followed by a vertical shift down 4 units, so $c(x) = \frac{1}{4}f(-x) - 4$.

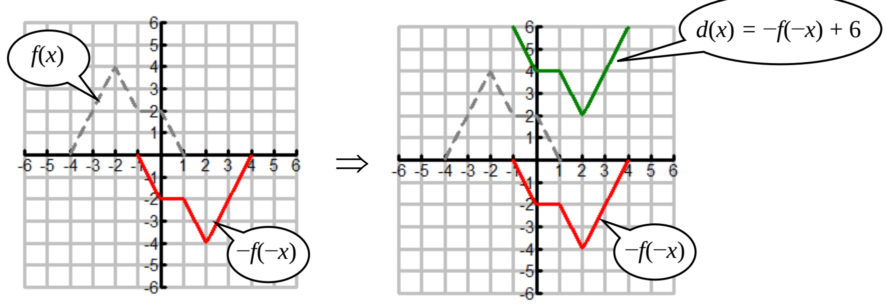


$f(-x)$ is a horizontal reflection of $f(x)$.

$\frac{1}{4}f(-x)$ is a vertical compression of $f(-x)$.

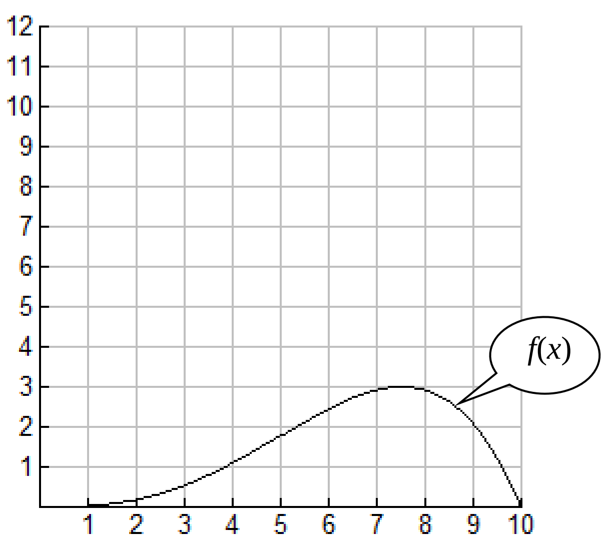
$\frac{1}{4}f(-x) - 4$ is a vertical shift of $\frac{1}{4}f(-x)$ down 4.

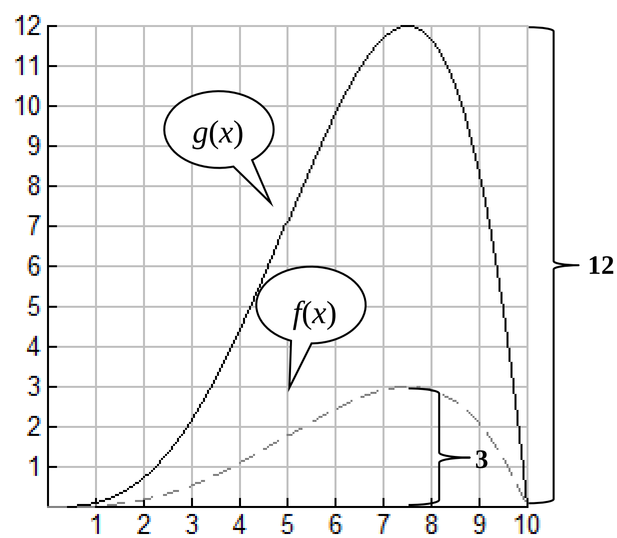
d. The graph of $y = d(x)$ is a horizontal and vertical reflection, followed by a vertical shift up 6. This is the graph of $b(x)$ shifted up 6 units.



4. The graph of $y = f(x)$.is shown. Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$. Find a formula for $g(x)$ in **<u>terms of $f(x)$.</u>**

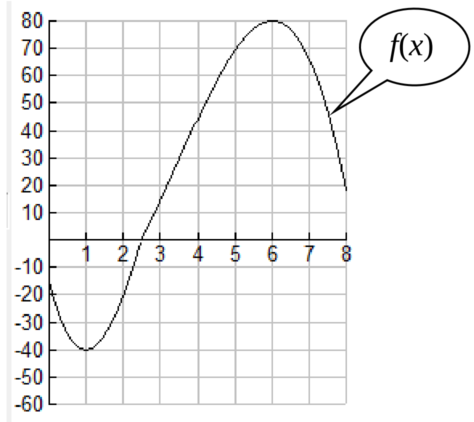
a.

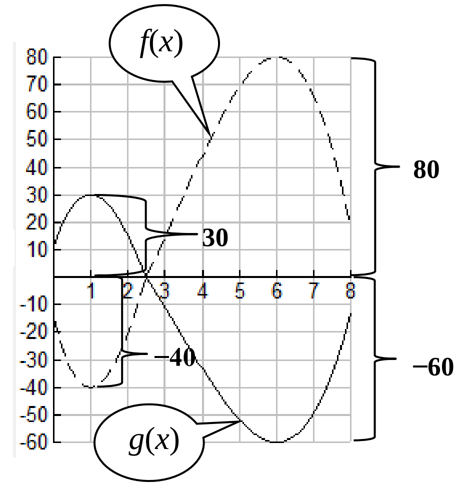




The outputs of $g(x)$ are *larger* than those for $f(x)$ so it is a *vertical stretch*. Compare maximum points. The graph of $g(x)$ is a vertical stretch of the graph of $f(x)$ by a factor of k, where $3k = 12$. Thus $k = 4$ and $g(x) = 4f(x)$.

b.





The outputs of $g(x)$ are *smaller* than those for $f(x)$ so it is a *vertical shrink*. Compare maximum points. The graph of $g(x)$ is a vertical compression of the graph of $f(x)$ by a factor of k, where $80k = -60$. You could also compare minimum points: $-40k = 30$. In either case, $k = -0.75$ and $g(x) = -0.75\, f(x)$.

5. Which of these is $\ln \sqrt[3]{x^2}$? Circle one.

A. $3\ln\sqrt{x}$ B. $3\ln x$ C. $x\ln 3$ D. $\frac{2}{3}\ln x$ E. $\frac{3}{2}\ln x$ F. $3\ln x^2$ G. $2\ln x^3$ H. None of these.

$\ln \sqrt[3]{x^2} = \ln x^{2/3} = \frac{2}{3}\ln x$ so Choice D.

6. Solve $e^x = 17.3$

EXACT: $x = \ln 17.3$

APPROXIMATE: $x \approx 2.85$

We have $e^x = 17.3$ — Since the base is e, take natural logarithms of both sides.

$\ln e^x = \ln 17.3$ — Use the inverse property $\ln e^Q = Q$

$x = \ln 17.3$

Check: If $x \approx 2.85$ and $e^x = 17.3$, then $e^{2.85} \approx 17.3$

7. **a.** $5\ln(3x) = 20$

EXACT: $x = \frac{1}{3}e^4$ or $\dfrac{e^4}{3}$

APPROXIMATE: $x \approx 18.199$

We have $5\ln(3x) = 20$ — Divide both sides by 5.

$\ln(3x) = 4$ — Make both sides a power of e.

$e^{\ln(3x)} = e^4$ — Use inverse property

$3x = e^4$ — Divide both sides by 3

$x = \frac{1}{3}e^4$ or $\dfrac{e^4}{3}$

Check: If $x \approx 18.199$ and $5\ln(3x) = 20$, then $5\ln(3\cdot 18.199) \approx 20$

b. $5\log x + 7 = 10$

EXACT: $x = 10^{3/5}$

APPROXIMATE: $x \approx 3.981$

We have $5\log x + 7 = 10$ Subtract 7 from both sides.

$5\log x = 3$ Divide both sides by 5.

$\log x = \frac{3}{5}$ Make both sides a power of 10.

$10^{\log x} = 10^{3/5}$ Use inverse property.

$x = 10^{3/5} \approx 3.981$

Check: If $x \approx 3.9815$ and $5\log x + 7 = 10$, then $\log(3.981) + 7 \approx 10$

8 a. $4u(u-2)=0$

This is a quadratic equation, but how nice! It is already in factored form.
This can be solved by setting each factor equal to 0 and
applying the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$.

We have $4u = 0$ and $u - 2 = 0$.
To solve $4u = 0$ we just divide both sides by 4 to get $u = 0$. To solve $u - 2 = 0$, add 2 to both sides.
The two solutions are $u = 0, 2$. (This can also be done just by inspection.)

A common error is to multiply these out to get $4u^2 - 8u = 0$.
This is similar to part d. But unfortunately that takes you into the wrong direction.
(Multiplying out is like returning clean clothes out of the dryer and putting them back in the washer. Oops!☺)
Since it is already factored, you do not want to reverse the factorization by distributing.

b. $25u^2 = 4$

This is a quadratic equation, but since it contains only u^2 we can divide by 25 and take square roots.

We have $25u^2 = 4$ Divide both sides by 25.

$u^2 = \frac{4}{25}$ Take square roots of both sides. Remember there are two square roots.

$$u = \pm\sqrt{\frac{4}{25}} = \pm\frac{2}{5}$$

Alternatively, you can get 0 on one side and factor, and
use the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$

$$25u^2 = 4$$

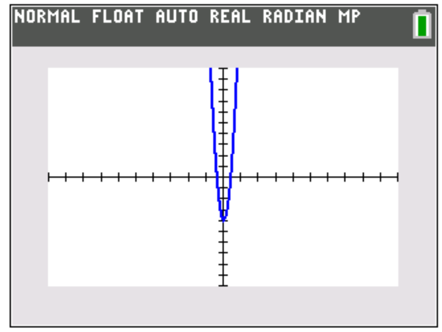
$$25u^2 - 4 = 0$$ Then set each factor equal to 0.

$$(5u+2)(5u-2) = 0$$

$5u + 2 = 0$ ‖ $5u - 2 = 0$

$u = -\frac{2}{5}$ ‖ $u = \frac{2}{5}$

A common error is to only report the positive solution and forget there is a negative square root.
A quick sketch of $y = 25x^2 - 4$ confirms there are two zeros.
A sketch may just be done with pencil and paper using knowledge of transformations.
The graph of $y = 25x^2 - 4$ is a vertical shift of the graph of $y = 25x^2$ down 4 units.
The graph of $y = 25x^2$ is a vertical stretch of the graph of $y = x^2$ by a factor of 25.



c. $25u^2 = 4u$
This is a quadratic equation.
Get 0 on one side of the equation so you can apply the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$.
Factor.

We have $25u^2 - 4u = 0$ Factor out the greatest common factor, which here is *u*.
$u(25u - 4) = 0$ Set each factor equal to 0 using the zero product property.

$$u = 0 \quad \Big\| \quad 25u - 4 = 0, \quad u = \tfrac{4}{25}$$

There are two solutions, namely $u = 0, \frac{4}{25}$.

A common error is to divide both sides of $25u^2 = 4u$ by *u*.
Don't do this. You will lose a solution.

d. $13x - 4x^2 = 0$
This is a quadratic equation similar to part c. Happily 0 is already on one side of the equation. Factor.

We have $13x - 4x^2 = 0$ Factor out the greatest common factor, which here is *x*.
$x(13 - 4x) = 0$ Set each factor equal to 0 using the zero product property.

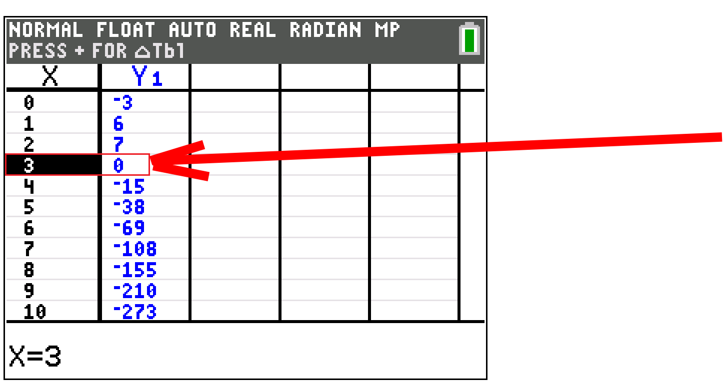
$$x = 0 \quad \Big\| \quad 13 - 4x = 0, \quad x = \tfrac{13}{4} \text{ or } 3.25$$

There are two solutions, namely $u = 0, \frac{13}{4}$.

e. $13x - 4x^2 = 3$
This is a quadratic equation. Get 0 on one side of the equation.

$13x - 4x^2 - 3 = 0$

Enter $Y_1 = 13X - 4X^2 - 3$ into a grapher. A table can quickly unveil if there are any integer zeros.



The table shows that one zero of $y = 13x - 4x^2 - 3$ is **3**.

Since 3 is a zero, then $(x - 3)$ is a factor of the equation.
Let's arrange terms in descending powers of x: $-4x^2 + 13x - 3 = 0$

Multiply both sides by -1 so that the quadratic term is positive. $4x^2 - 13x + 3 = 0$
This will make it easier to work with.

$(\_\_x \;-\; \_\_\_)(x - 3) = 4x^2 - 13x \; \color{red}{+\; 3} \; = 0$

LAST TERMS

Ask $(\boxed{?}) \cdot (-3) = 3$. The product of both of these is **3** so we have –1 and –3.

$(\_\_x \;-\; \underline{\;1\;})(x - 3) = \color{red}{4x^2} - 13x \; + \; 3 \; = 0$

FIRST TERMS

Ask $(\boxed{?}) \cdot (x) = 4x^2$. The product of both of these is **$4x^2$** so we have $4x$ and x.

Thus we have $(4x - 1)(x - 3) = 0$. Use the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$
Set each factor equal to 0 and solve.

$$(4x - 1)\times(x - 3) = 0$$

$4x - 1 = 0$	$x - 3 = 0$
$x = \frac{1}{4}$	$x = 3$

The solutions are $x = \frac{1}{4}, 3$.

You can also find the zeros using the graph of $Y_1 = 13X - 4X^2 - 3$ or
This can also be confirmed with a table if the increment is set to 0.25.

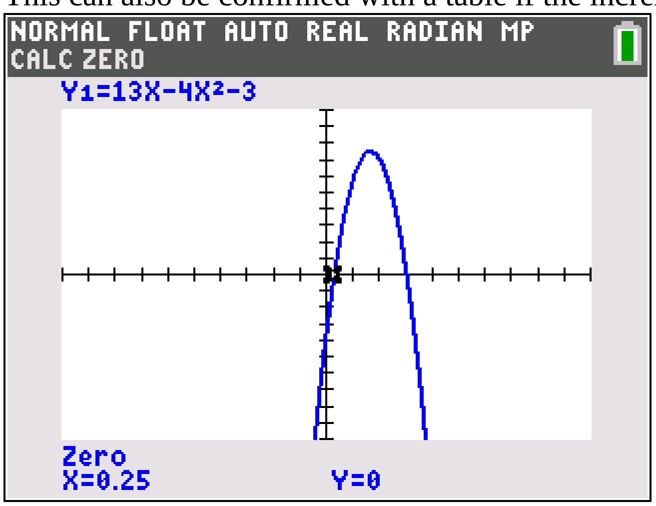


TABLE SETUP
TblStart=0
ΔTbl=.25
Indpnt: Auto Ask
Depend: Auto Ask

X	Y_1
0	-3
0.25	0
0.5	2.5
0.75	4.5
1	6
1.25	7
1.5	7.5
1.75	7.5
2	7
2.25	6
2.5	4.5
2.75	2.5
3	0

8. **f.** $2u^2 = u + 1$

Since this is a quadratic equation with three terms, get 0 on one side of the equation.

$2u^2 - u - 1 = 0$

We have a trinomial. One way to solve this equation is to try to factor.

(___ + ___)(___ + ____) = $2u^2 - u - 1 = 0$

FIRST TERMS — The product of both of these is $2u^2$ so we have $2u$ and u in the blanks.

($2u$ + ___)(u + ____) = $2u^2 - u - 1 = 0$

LAST TERMS — The product of both of these is −1 so we have 1 and −1 in the blanks. But which goes where?

The sum of the product of the inner and the product of the outer is the middle term $-u$.
One of u and $2u$ must be negative, and the sum must be $-u$.
This is only possible if $2u$ is negative and u is positive. This means $2u$ is multiplied by −1.

($2u$ 1)(u 1) = $2u^2 - u - 1 = 0$

u

$+ -2u$

$-u$

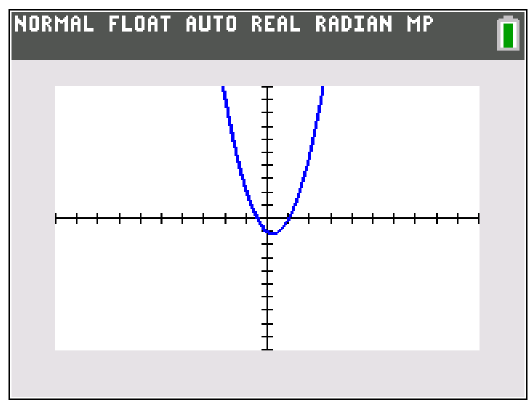
Thus we have $(2u + 1)(u - 1) = 0$. Use the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$

Now set each factor equal to 0 and solve. $(2u + 1)\times(u - 1) = 0$

$2u + 1 = 0$	$u - 1 = 0$
$u = -\frac{1}{2}$	$u = 1$

The solutions are $u = -\frac{1}{2}, 1$.

The solution process can be enhanced by using a grapher.
In the Y= Editor, enter Y₁=2X²−X−1.
The zeros of the graph appear to be 1 and $-\frac{1}{2}$.



The zeros of $y = 2x^2 - x - 1$ are the solutions to $2x^2 - x - 1 = 0$

This can be confirmed with a table:

TABLE SETUP
TblStart=-0.5
△Tbl=.5
Indpnt: Auto Ask
Depend: Auto Ask

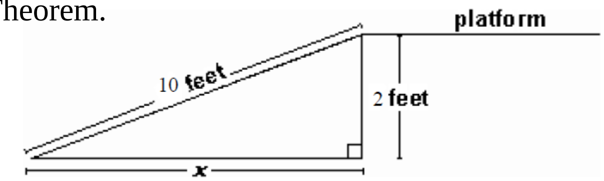
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y_1
-0.5	0
0	-1
0.5	-1
1	0
1.5	2
2	5
2.5	9
3	14
3.5	20
4	27
4.5	35

X=-0.5

9. Use the Pythagorean Theorem.

$$x^2 + 2^2 = 10^2$$
$$x^2 + 4 = 100$$
$$x^2 = 96$$
$$x = \pm\sqrt{96}$$



Since x is the length of a side, it is the positive square root $\sqrt{96}$ and Choice C is correct.

10. **a**. Choice B is true.
Linear functions grow by a constant rate, and
exponential functions grow by a constant percent rate.

b. Jonesville: $P = 5000 + 200t$

c. Smithville: $P = 5000(1.02)^t$

11. **a.** The function P is exponential. $P = 200(1.23)^x$.
The function Q is linear. $Q = 400 + 200x$

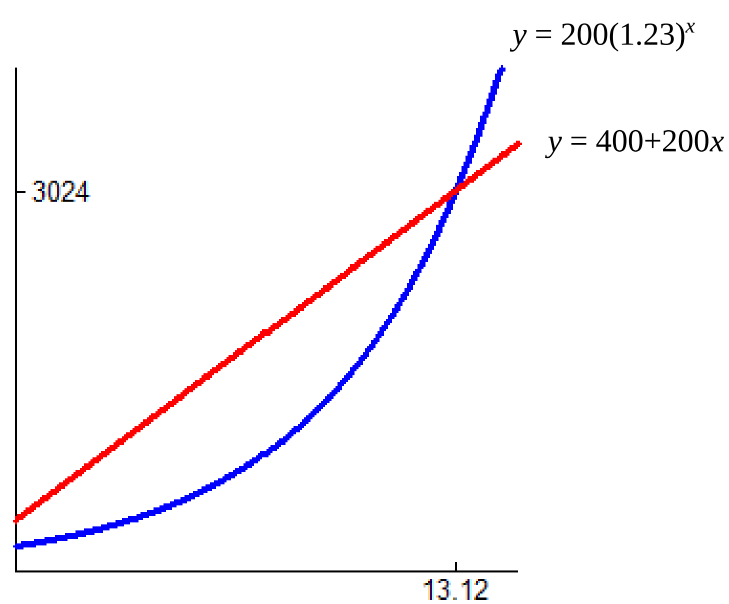
b. $x \approx 13.12$ years

The equation $200(1.23)^x = 400 + 200x$ is not possible to solve algebraically.

Method 1: Using a table, enter the formulas Y1= **200(1.23)^x** **and** Y2 = **400 + 2x** in Y= and scroll. Eventually set your step size to 0.01

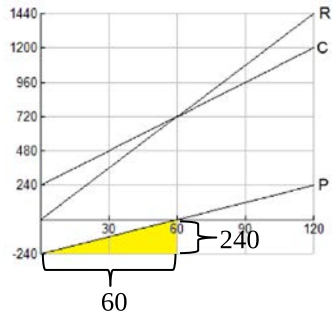
X	Y1	Y2
13.1	3011.5	3020
13.11	3017.8	3022
13.12	3024	3024
13.13	3030.3	3026
13.14	3036.6	3028

Method 2: Using a graph, set a viewing window (aided by the table) and then find the intersection point.

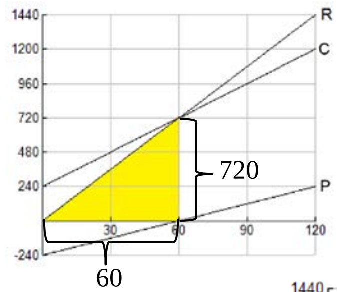


12. **a.** The x-coordinate of the intersection point of R and C is $x = 60$.

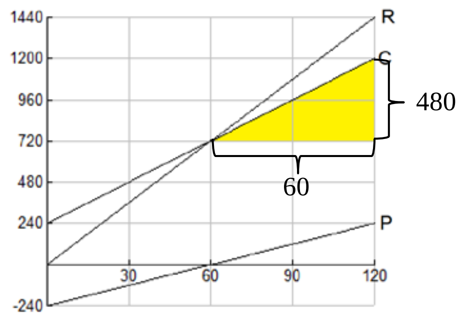
b. The slope of $P(x)$ is $\frac{\Delta y}{\Delta x} = \frac{240}{60} = 4$.
The vertical intercept is (0, −240).
So $P(x) = 4x - 240$.



The slope of $R(x)$ is $\frac{\Delta y}{\Delta x} = \frac{720}{60} = 12$.
The vertical intercept is (0, 0).
So $R(x) = 12x$.



The slope of $C(x)$ is $\frac{\Delta y}{\Delta x} = \frac{480}{60} = 8$.
The vertical intercept is (0, 240).
So $C(x) = 8x + 240$.



Another method: Since $P = R - C$,
you can also find C once you know P and R.
$P + C = R$ so $4x - 240 + C = 12x$
Add 240 to both sides and subtract $4x$ from both sides.
Thus $C = 8x + 240$.

13. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.
Suppose that you have \$6000 to invest. Which investment yields the greater return over 13 years:
8.07% compounded **continuously** or 8.1% compounded **monthly**?

\$6000 at 8.07% compounded **continuously** for 13 years returns $A = 6000e^{0.0807\times 13} \approx \$17,130.48$
\$6000 at 8.1% compounded **monthly** for 13 years returns $A = 6000(1+\frac{.081}{12})^{12\times 13} \approx \$17,136.69$
Choice B. Investing \$6000 at 8.1% compounded **monthly** over 13 years yields the greater return.

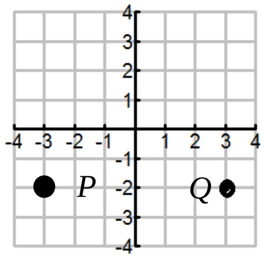
14. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.
Suppose that you have \$9000 to invest. Which investment yields the greater return over 19 years:
7.88% compounded **continuously** or 7.9% compounded **monthly**? (Select one)
\$9000 at 7.88% compounded **continuously** for 19 years returns $A = 9000e^{0.0788\times 19} \approx \$40,222.42$
\$9000 at 7.9% compounded **monthly** for 19 years returns $A = 9000(1+\frac{.079}{12})^{12\times 19} \approx \$40,177.43$
Choice A. Investing \$9000 at 7.88% compounded **continuously** over 19 years yields the greater return.

15. Suppose the point $P(3,-2)$ is a point on the graph of $y = f(x)$

a. Suppose $f(x)$ is **even:**

i. Report the coordinates of another point Q, which corresponds to P. (-3 , -2)

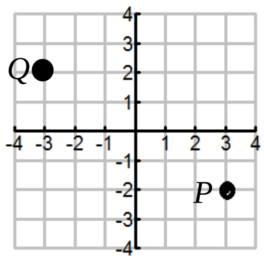
ii. Plot the point Q on the grid provided.



b. Suppose $f(x)$ is **odd**:

i. Report the coordinates of another point Q, which corresponds to P. (-3 , 2)

ii. Plot the point Q on the grid provided.



16. a. Assume the table represents a **linear** function.

i. Complete the box in the first row and the last row.

For a linear function, the difference of consecutive outputs is constant. We see from the table that difference is **−40**. Therefore the output when $x = 4$ is $45 - 40 =$ **5**. Similarly, the output when $x = 0$ is $125 + 40 =$ **165**.

x	y	
0	**165**	
1	125	$= (\boxed{?}) - 40$
2	85	$= 125 - 40$
3	45	$= 85 - 40$
4	**5**	$= 45 - 40$

ii. Report the **equation** of the line in slope-intercept form: $y =$ **−40** $x +$ **165**

b. Assume the table represents an **exponential** function.

i. Complete the box in the first row and the last row.

For an exponential function, the ratio of consecutive outputs is constant. We see from the table that ratio is **1/4**. Therefore the output when $x = 4$ is $8 \times \frac{1}{4} =$ **2**. Similarly, the output when $x = 0$ is $128 \times 4 =$ **512**.

x	y	
0	**512**	
1	128	$= (\boxed{?}) \times \frac{1}{4}$
2	32	$= 128 \times \frac{1}{4}$
3	8	$= 32 \times \frac{1}{4}$
4	**2**	$= 8 \times \frac{1}{4}$

ii. If we report the equation of the exponential function in the form $y = ab^x$, then we have

$a =$ **512** and $b =$ $\mathbf{\frac{1}{4}}$